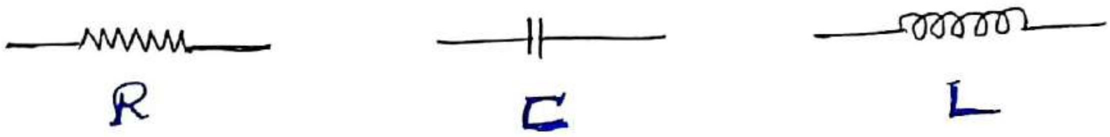
# ELECTRICAL ANALYSIS

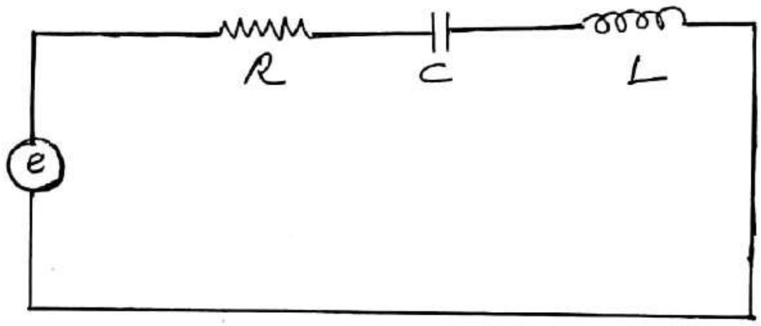
## Basic circuit Components.

* The Basic circuit components are Resistors, capacitors and Inductors.




## Network

* The Combination of R, L, C along with various electrical Sources called as "electrical circuits" (or) "Networks."




* Any individual ckt elements with two terminals which can be connected to other ckt element is known as "Network element".
* Generally the network elements can classify as two types such as 1. Active elements 2. Passive elements.
  - Active elements : Voltage source, Current source
  - Passive elements : R, L, C.

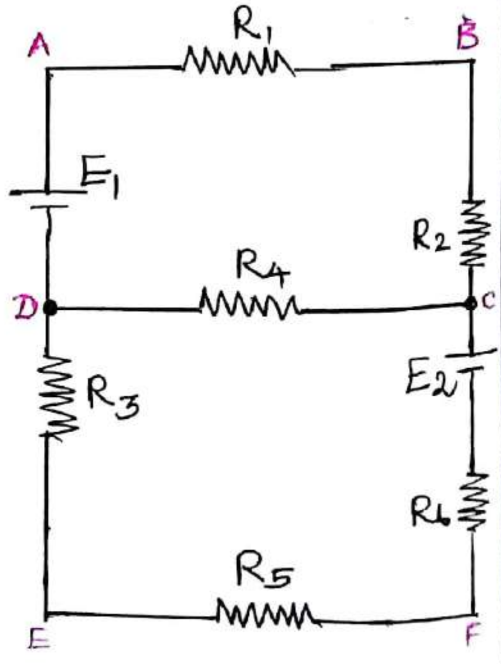
* Which connects the Various points of the network with one another is called a "Branch". A branch may contain more than one element.

* The meeting point of three or more branches is called as "Junction Point".

* The connecting point of two or more elements is called as "Node". All junction points of the network also called as "Node".

* A closed path in a network which formed by a set of branches is called "Mesh". In a mesh, suppose any one of branch is removed then the remaining branches do not form a closed path.

* Each mesh consider as smallest loop. But the loop may or may not be a mesh.




→ mesh : A B C D A.

→ Loop : A B C F E D A.

## Classification of Electrical Network.

* Based on the charecteristics of Network, the Network can classify as follow,
  - Linear Network.
  - Non Linear Network.
  - Bilateral Network.
  - Unilateral Network.
  - Active Network.
  - Passive Network.
  - Lumped Network.
  - Distributed Network.

Network charectoristics depend on the charectoristics of its elements such as R, L and C.

### 1. Linear Network.

* Network chanecteristics are always constant, irrespective of the Variations in time, Voltage and temperature etc. Such Network is called "Linear Network".
* Ohm's law and superposition law can be applied.

### 2. Non Linear Network.

* Network chareeteristics are dependent of the Variations in Voltage, time, and temperature etc. Such Network called as "Non Linear Network".
* ohm's law maynot be applied.
* Supper position law can not be applied.
* ex. Diode ⇒ Diode current varies with voltage Vatiations.

## 3. Bilateral Network.

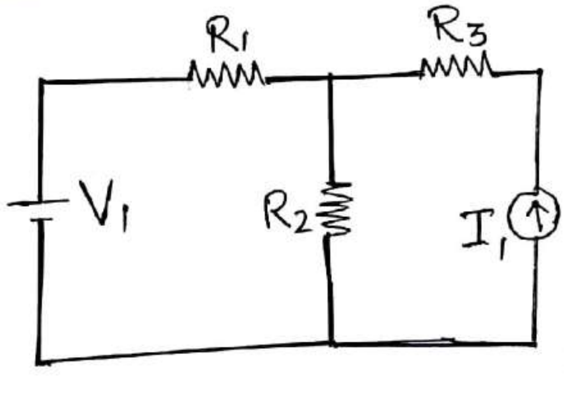
* Network charecteristics are independent of current flow direction.
* Example: Network which consists only R.

## 4. Unilateral Network.

* Network charecteristics are dependent of current flow direction.
* Example: Network which consists Diode also.

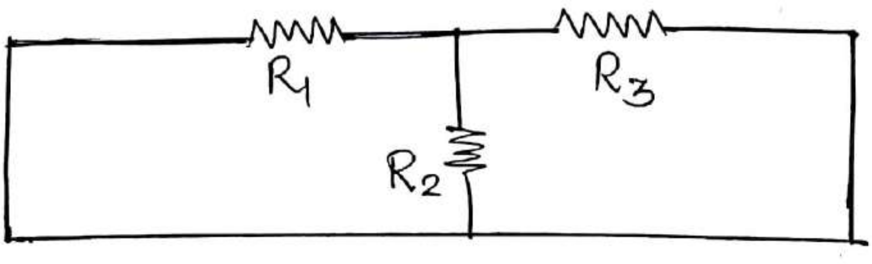
## 5. Active Network.

* A Network which consists atleast one source of energy.
* The energy source may be voltage or current.




## 6. Passive Network.

* A Network which doesn't have any kind of energy source.



## Enery Sourcos.

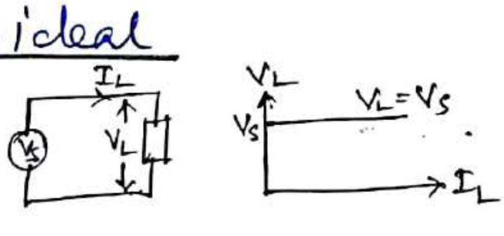
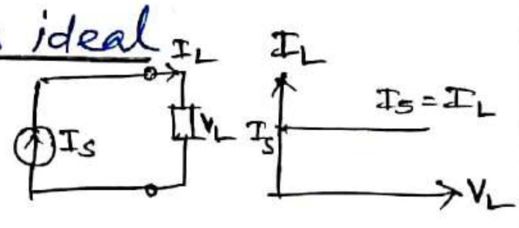
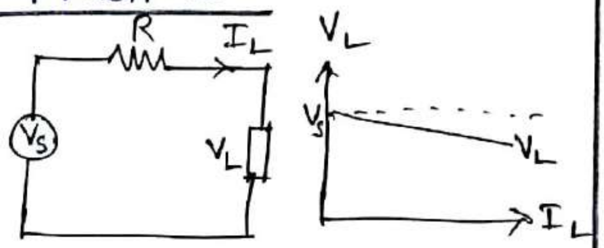
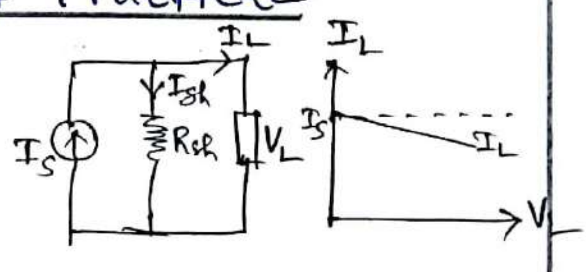
* Generally classified as,
  1. Ideal Source
  2. Practical Source.

### 1. Ideal Source.

* Gives Constant energy and independent of another parameter.

### 2. Practical Source.

* Practically every Voltage and Current Sources have some minimum amount of internal Resistance value. So it is not possible to give constant energy and dependent on another - parameters.

| ⇒ Voltage Source. | ⇒ Current Source. |
|---|---|
| * ideal  | * ideal  |
| * Practical.  | * Practical  |

## SERIES CIRCUIT

* Several resistors are connected one after the other.
* Current flow remains same in all resistors and the entire circuit.
* Voltage across each resistor is differed.

$V_1 \neq V_2 \neq V_3 \neq V_4$

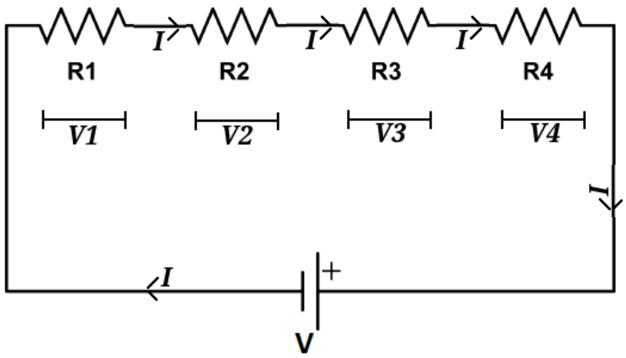
* The input voltage to the circuit is equal to the sum of voltage drop across each resistor.

$V = V_1 + V_2 + V_3 + V_4$

* The total resistance in the circuit is,

$R_T = R_1 + R_2 + R_3 + R_4$

* The total resistance ($R_T$) $R_T > R_1$ ; $R_T > R_2$ ; $R_T > R_3$ ; $R_T > R_4$




## PARALLEL CIRCUIT

* Several resistors are connected across to one another.
* Voltage drop remains same in all resistors and the entire circuit.
* Current flow in each resistor is differed.

$I_1 \neq I_2 \neq I_3 \neq I_4$

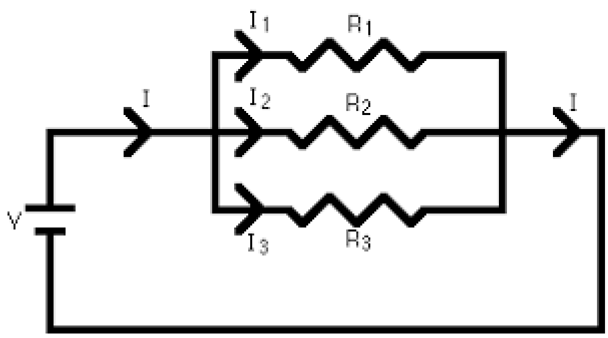
* The input current to the circuit is equal to the sum of current flow in each resistor.

$I = I_1 + I_2 + I_3 + I_4$

* The total resistance ($R_T$) in the circuit is,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

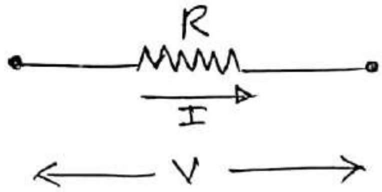
* The total resistance ($R_T$) $R_T < R_1$ ; $R_T < R_2$ ; $R_T < R_3$



## Ohm's law.

* Voltage across the electric circuit is direct proportional to the product of flow of Current and Resistance of the same circuit, provided that the temperature of the circuit (or) Conductor remains constant.

$$V = IR$$



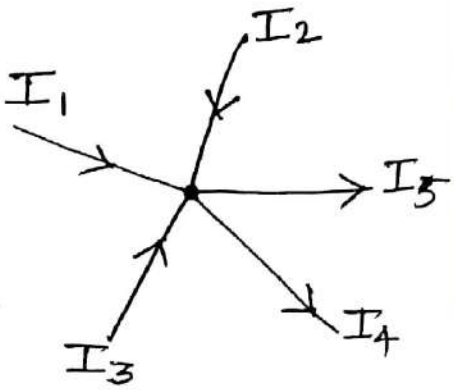

* Limitations.
    1. Not applicable to the nonlinear devices such as Diodes, Zener Diodes, voltage regulators.

## Kirchoff's law.

### 1. Kirchoff's Current law [KCL].

* The sum of current flowing towards a Junction point is equal to the sum of current flowing away from the same Junction point.

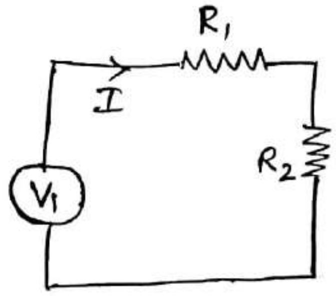
$$I_1 + I_2 + I_3 = I_4 + I_5$$



## 2. Kirchoff's Voltage law (KVL)

* The algebraic sum of all the branch Voltages, around any closed path is always zero.

$$V_1 + IR_1 + IR_2 = 0$$




## Electric Power.

### 1. Instantaneous Power.

* Instantaneous power is the power at any instant of time.
* It is the product of time functions of the Voltage and Current
* Unit of instantaneous power is "VA".

$$P(t) = V(t) \cdot I(t) \quad VA.$$

### 2. Average Power.

* It is the average of the instantaneous power over one instant (or) time period.

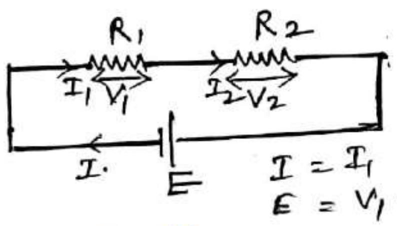
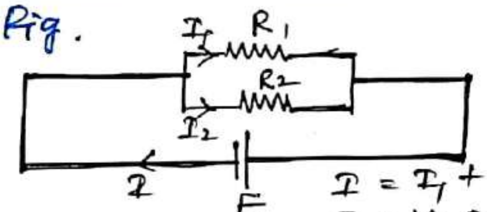
$$P_{avg} = VI \cos\phi.$$

// $\cos\phi$ = Power factor.

# Nodal Analysis.

* Steps to be followed for Nodal Analysis.

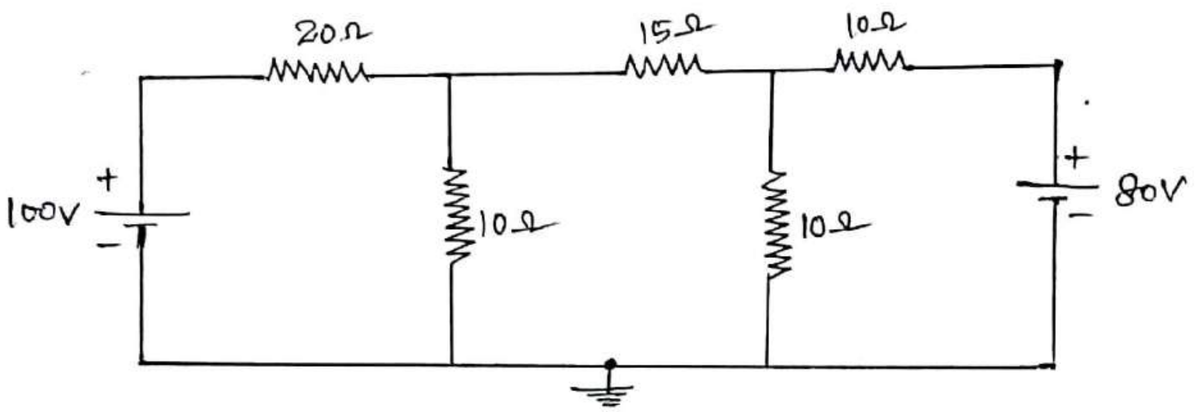
1. choose the nodes and show the node Voltages.
2. Show the Vareous branch currents with flow direction.
3. Apply KCL at nodes.
4. Derive the expressions for branch Currents.
5. By solving the derived current expressions the requird electrical parameters can be found.

| Series circuit | Parallel circuit. |
| --- | --- |
| * Fig.   $I = I_1 = I_2$<br>$E = V_1 + V_2$ | * Fig.   $I = I_1 + I_2$<br>$E = V_1 = V_2$ |
| * Current flow remains Constant in all Resists. | * Each resistors having different Current flow. |
| * Each Resistors having different voltage across themselves. | * Each Resistors having Same Voltage across themselves |
| * All branch current will be equal to the net circuit current. | * Voltage across each resistor will be equal to the voltage across the net circuit. |

# Tutorials on Nodal Analysis.

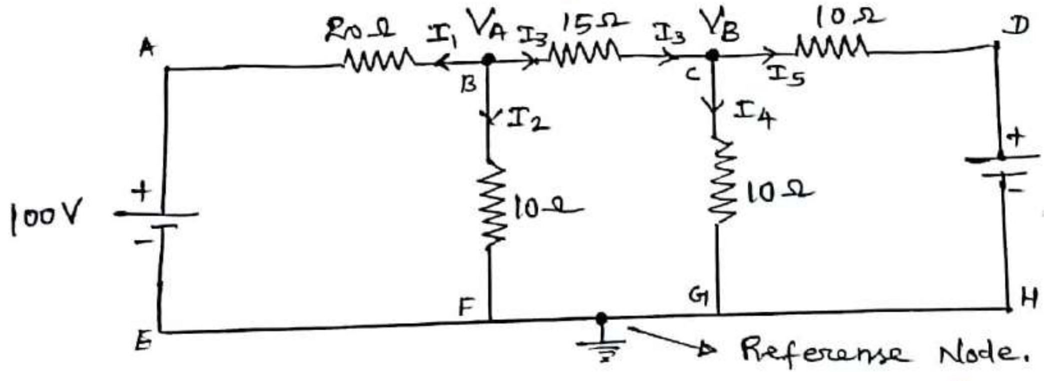
**ex.1**

Calculate the voltage across the 15 Ω resistor in the given network by using Nodal analysis. (May-2005)




**Sol**

* Fix the node voltages and redraw the diagram.




* Node A and B are the "major nodes".
* Apply KCL at node B and C.

→ at node B ⇒ $-I_1 - I_2 - I_3 = 0$

$$I_1 + I_2 + I_3 = 0 \longrightarrow (1)$$

→ at node C ⟶ $I_3 - I_4 - I_5 = 0 \longrightarrow (2)$

* The expressions for various branch currents are

$$I_1 = \frac{V_A - 100}{20} = 0.05V_A - 5 \longrightarrow (3)$$

$$I_2 = \frac{V_A}{10} = 0.1\, V_A \longrightarrow (4)$$

$$I_3 = \frac{V_A - V_B}{15} = 0.066 V_A - 0.066 V_B \longrightarrow (5)$$

$$I_4 = \frac{V_B}{10} = 0.1\, V_B \longrightarrow (6)$$

$$I_5 = \frac{V_B - 80}{10} = 0.1 V_B - 8 \longrightarrow (7)$$

* Now put all current values in eq-no. (1), (2)

$$(1) \Rightarrow I_1 + I_2 + I_3 = 0$$

$$[0.05 V_A - 5] + 0.1 V_A + [0.066 V_A + 0.066 V_B] = 0$$

$$0.2166 V_A - 0.066 V_B = 5 \longrightarrow (8)$$

$$(2) \Rightarrow I_3 - I_4 - I_5 = 0$$

$$[0.066 V_A - 0.066 V_B] - 0.1 V_B - [0.1 V_B - 8] = 0$$

$$-0.066 V_A + 0.2666 V_B = 8 \longrightarrow (9)$$

* By solving eq. (8) and (9)

$$V_A = -34.99 = 35 \text{ Volts}$$

$$V_B = -38.75 = 38.75 \text{ Volts.}$$

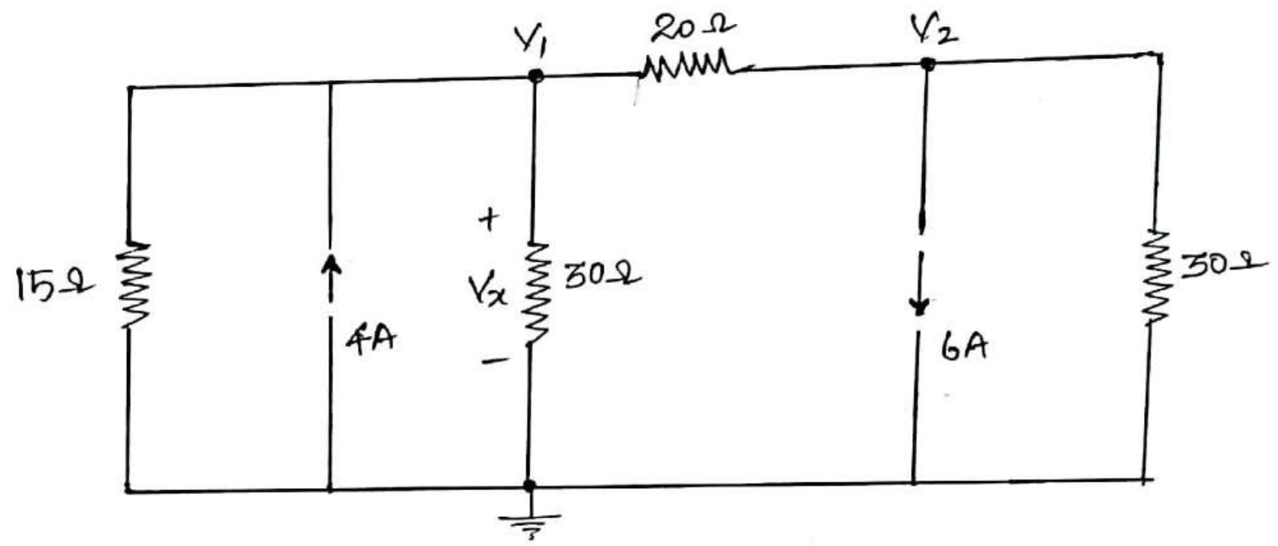
$$V_{AB} = V_B - V_A = 38.75 - 35$$

$$V_{AB} = 3.75 \text{ Volts With B positive.}$$

* Voltage across 15 Ω Resistor ⇒ 3.75 Volts.

⇒ Voltage across 15 Ω Resist. ⇒ $V_{15\Omega} = I_3 * R = 3.75$ Volts.

Ex. No: 2. For the given ckt determine the value of $V_x$ using Nodal Analysis.

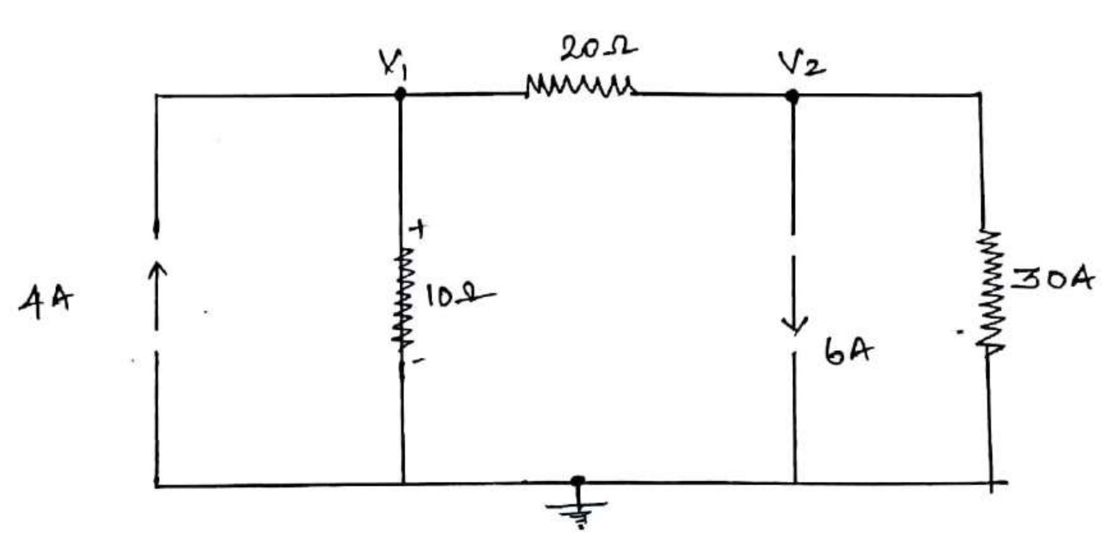



Sol

* In the above circuit Resistances 15 Ω and 30 Ω are connected parallel along with 4 A source and themselves also. So the equivalent Resistance

$$R_{eq} = \frac{15\,\Omega * 30\,\Omega}{15\,\Omega + 30\,\Omega} = 10\,\Omega$$

* Now the above circuit can be redrawn as follows.



* at node - 1.

$$\frac{V_1}{10} + \frac{V_1 - V_2}{20} = 4$$

$$\frac{V_1}{10} + \frac{V_1}{20} - \frac{V_2}{20} = 4$$

$$\left(\frac{1}{10} + \frac{1}{20}\right)V_1 - \frac{V_2}{20} = 4$$

$$\frac{3}{20} V_1 - \frac{V_2}{20} = 4$$

$$3V_1 - V_2 = 80 \longrightarrow 1$$

* at node - 2

$$\frac{V_2 - V_1}{20} + \frac{V_2}{30} = -6$$

$$\frac{V_2}{20} - \frac{V_1}{20} + \frac{V_2}{30} = -6$$

$$-\frac{V_1}{20} + \left(\frac{1}{20} + \frac{1}{30}\right)V_2 = -6$$

$$-\frac{V_1}{20} + \left(\frac{50}{30 * 20}\right)V_2 = -6$$

$$-\frac{V_1}{20} + \left(\frac{50}{600}\right)V_2 = -6$$

$$-\frac{V_1}{20} + \frac{V_2}{12} = -6$$

$$\frac{(-12V_1 + 20V_2)}{20 * 12} = -6$$

$$(1/4) * (-12V_1 + 20V_2 = -6 * 240)$$

$$(-1) * (-3V_1 + 5V_2 = -6 * 60)$$

$$3V_1 - 5V_2 = 360 \longrightarrow 2$$

* By Solving eq (1) and (2) we can get the values of $V_1$, $V_2$

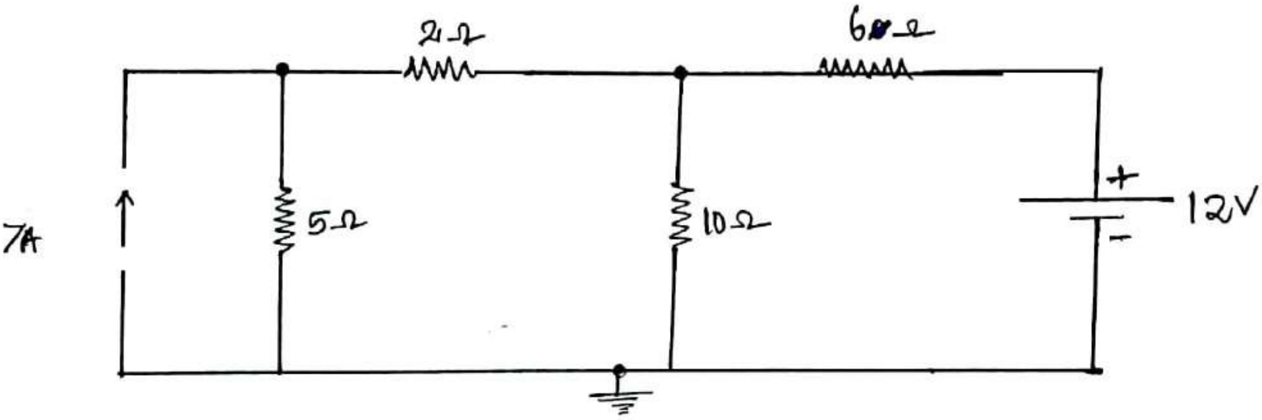
$V_1 = 3.33$ Volts.

$V_2 = -70$ Volts.

* From the given ckt. diagram $V_x = V_1$

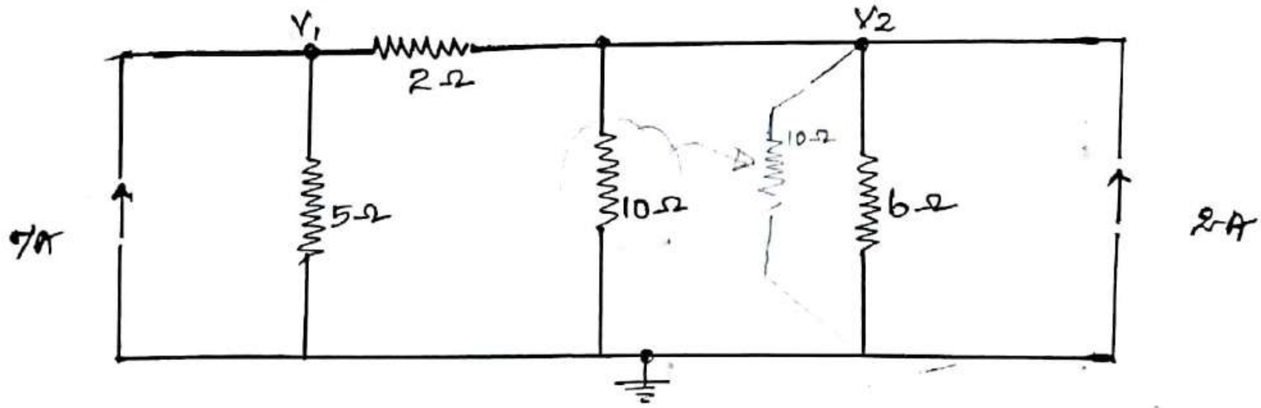
$V_x = 3.33$ Volts.

Ex.No. 3. Determine the Nodal Voltages and current through $2\,\Omega$ Resistance for the given ckt.




Sol

* Convert voltage source into current source and the modified circuit is given below.




* at node 1.

$$\frac{V_1}{5} + \frac{V_1 - V_2}{2} = 7$$

$$0.7V_1 - 0.5V_2 = 7 \rightarrow (1)$$

* at node - 2.

$$\frac{(V_2 - V_1)}{2} + \frac{V_2}{6} + \frac{V_2}{10} = 2$$

$$-0.5V_1 + 0.767V_2 = 2 \rightarrow (2)$$

* By Solving Eq ① and ② we can get the values for $V_1$ and $V_2$. [Called as nodal Voltages]

$$V_1 = 22.19 \text{ Volts.}$$

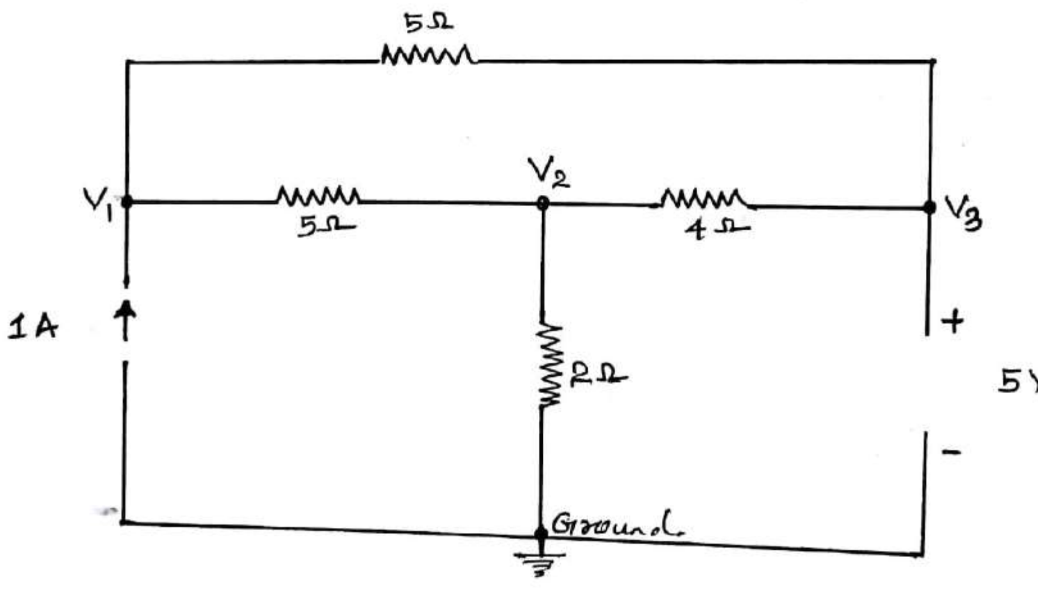
$$V_2 = 17.08 \text{ Volts.}$$

* From the Nodal Voltages can calculate the Current through $2\Omega$ Resist.

$$I_{2\Omega} = \frac{V_1 - V_2}{2} = \frac{(22.19 - 17.08)}{2}$$

$$I_{2\Omega} = 2.55 \text{ Amps.}$$

Ex.No. 4. Find the nodal Voltages for the given circuit and also find the power input and power output




Sol

* In this circuit no need to Convert the Voltage Source (5V) into Current source. Because Voltage Source (5V) is directly Connected between ground and $V_3$ and there is no anyother Resistor in Series with 5V. So we can Consider 5V Source as $V_3$ Node Voltage.

* Apply Nodal analysis At node $V_1$

* Current towards node ⇒ (+)
* Current outwards from node ⇒ (−)

$$\frac{(V_1 - V_2)}{5} + \frac{V_1 - V_3}{5} = +1$$

Where $V_3 = 5$ volts.

$$\frac{V_1}{5} - \frac{V_2}{5} + \frac{V_1}{5} - \frac{5}{5} = 1$$

$$V_1\left(\frac{1}{5} + \frac{1}{5}\right) - \frac{V_2}{5} - 1 = 1$$

$$\frac{2V_1}{5} - \frac{V_2}{5} = 1+1$$

$$2V_1 - V_2 = 10 \longrightarrow (1)$$

* Apply Nodl analysis at node $V_2$

$$\frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{4} + \frac{V_2}{2} = 0$$

$$\frac{V_2}{5} - \frac{V_1}{5} + \frac{V_2}{4} - \frac{V_3}{4} + \frac{V_2}{2} = 0$$

$$\frac{-V_1}{5} + V_2\left[\frac{1}{5} + \frac{1}{4} + \frac{1}{2}\right] - \frac{5}{4} = 0$$

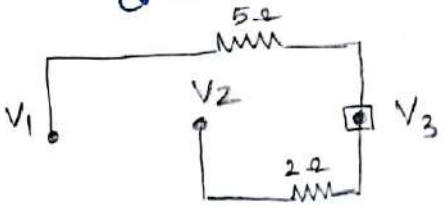
Where $V_3 = 5$ Volts.

$$-0.2\,V_1 + 0.95\,V_2 = 1.25 \longrightarrow (2)$$

* By solving eq. (1) and (2), We can get the nodal voltages of $V_1$ and $V_2$.

$V_1 = 6.32$ Volts.

$V_2 = 2.65$ Volts.




* output from the 1A current source.

$$P_c = V_1 * I = 6.32 * 1 = 6.32 \text{ Watts.}$$

* current from the Voltage Source

$$I_V = \frac{V_3 - V_2}{2} + \frac{V_3 - V_1}{5}$$

$$= \frac{5 - 2.65}{2} + \frac{5 - 6.32}{5}$$

$$= 1.175 + (-0.264)$$

$I_v = 0.911$ Amps.

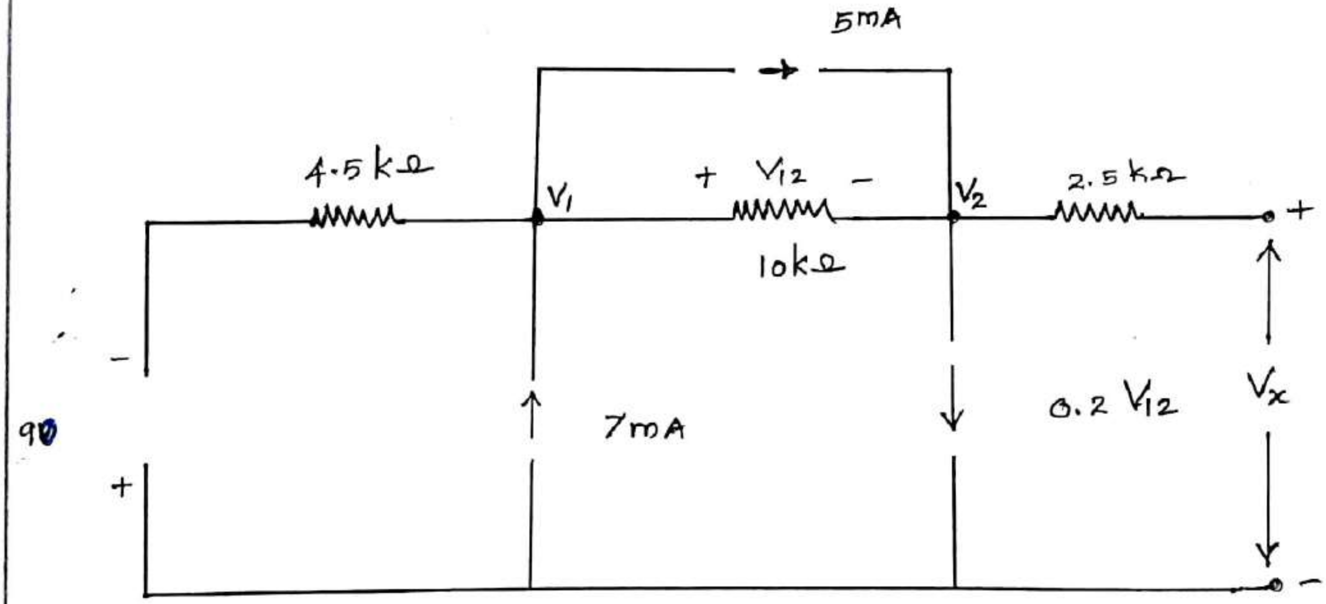
* output power from the Voltage Source.

$$P_v = V_s * I.$$

$$= 5 * 0.911$$

$$P_v = 4.555 \text{ Watts.}$$

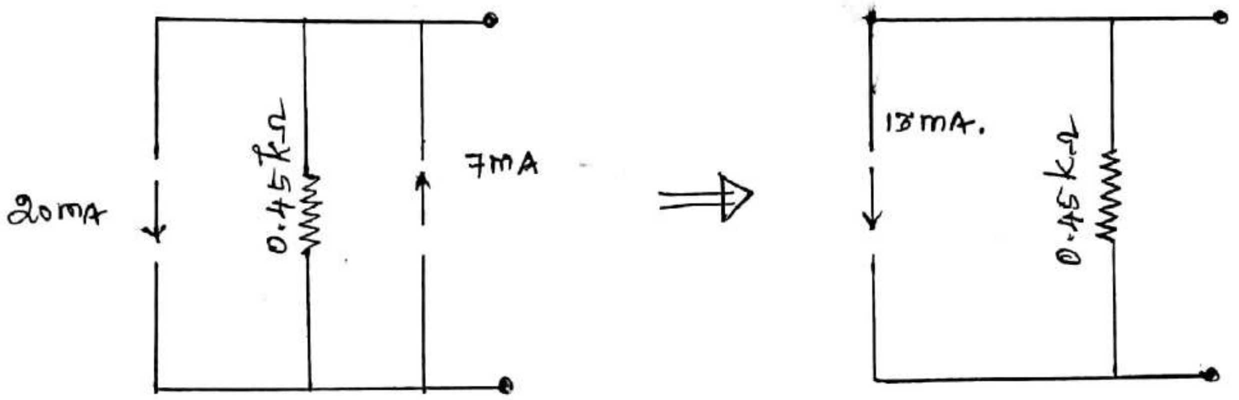
Ex. No: 5. Using Nodal Analysis determine $V_x$ in the given circuit.




Soln

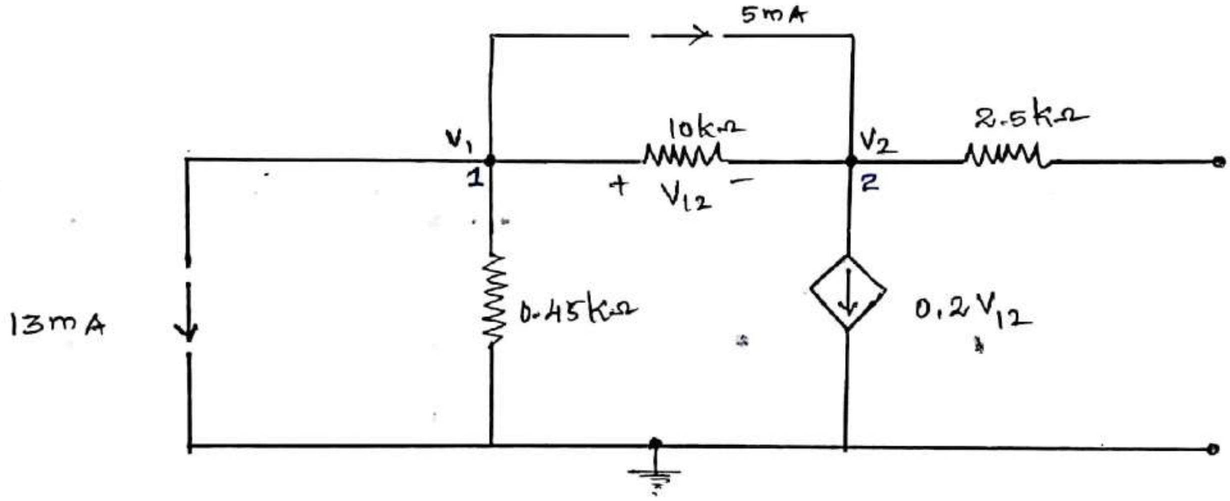
* Convert the Voltage source of 90V to Current source

$$I \Rightarrow \frac{90}{4.5k\Omega} = 20 \text{ mA.}$$




* The resultant current will be (20 mA − 7 mA) = 13 mA.

* Now redraw the new circuit.




* Apply Nodal analysis at node-1.

$$\frac{V_1}{0.45} + \frac{V_1 - V_2}{10} = -13 - 5$$

$$\frac{V_1}{0.45} + \frac{V_1}{10} - \frac{V_2}{10} = -18$$

$$2.32 V_1 - 0.1 V_2 = -18 \longrightarrow (1)$$

* Apply Nodal Analysis at node-2.

$$\frac{V_2 - V_1}{10} = 5 - 0.2 V_{12}$$

Where, $V_{12} = (V_1 - V_2)$

$$0.1 V_2 - 0.1 V_1 = 5 - 0.2 (V_1 - V_2)$$

$$-0.1V_1 + 0.1V_2 + 0.2V_1 - 0.2V_2 = 5$$

$$0.1V_1 - 0.1V_2 = 5 \longrightarrow (2)$$

* By Solving eq (1),(2) We can get the Node Voltages,

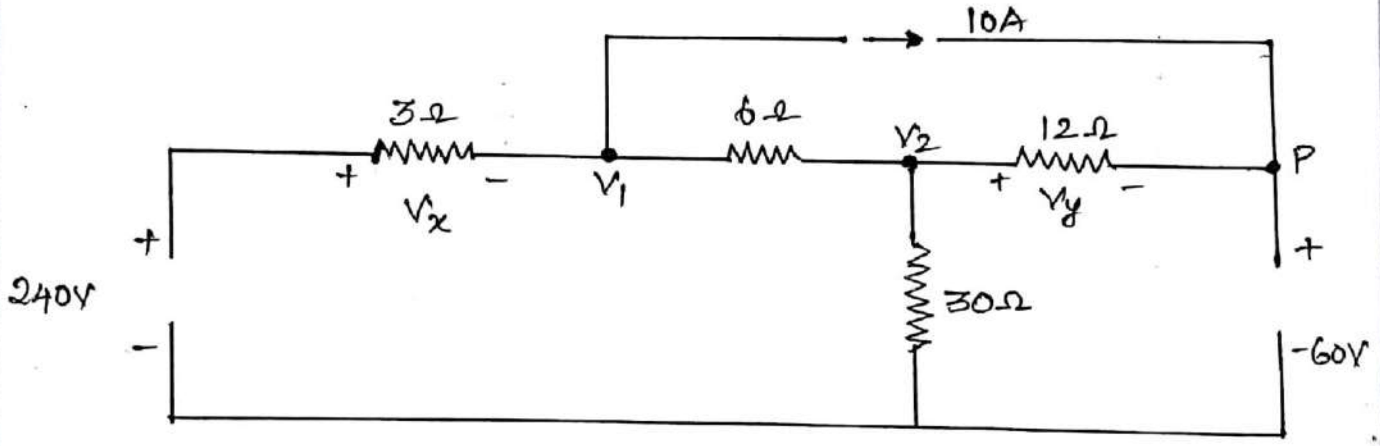
$$V_1 = -10.36 \text{ Volts.}$$

$$V_2 = -60.36 \text{ Volts.}$$

* Given circuit is open. So $V_x = V_2$

$$V_x = -60.36 \text{ Volts.}$$

Ex. No: 6. Determine $V_x$ and $V_y$ for the given circuit. And also find the power consumed by $6\Omega$.

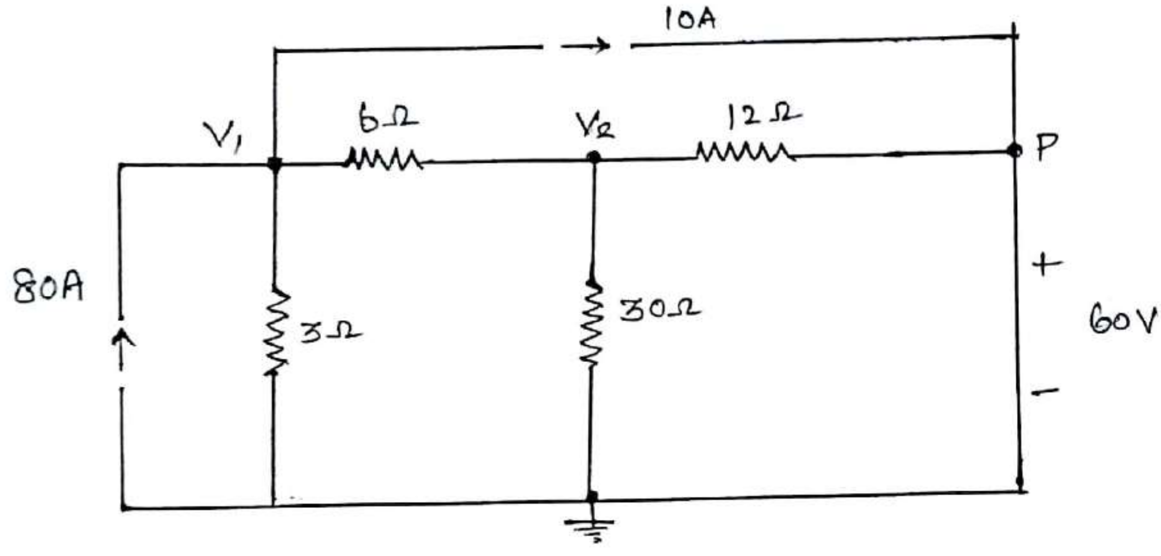



Sol

* At node P the voltage is fixed at 60 volts.
* $V_1$ and $V_2$ are independent nodes.
* Convert the Voltage Source 240V into Current Source.

$$\text{The new current source} = \frac{240\,V}{3\Omega}$$

$$= 80 \text{ Amps.}$$




* At node $V_1$

$$\frac{V_1}{3} + \frac{V_1 - V_2}{6} = 80 - 10$$

$$\frac{V_1}{3} + \frac{V_1}{6} - \frac{V_2}{6} = 70$$

$$0.5V_1 - 0.167V_2 = 70 \longrightarrow ①$$

* At node $V_2$

$$\frac{V_2 - V_1}{6} + \frac{V_2 - V_P}{12} + \frac{V_2}{30} = 0$$

$$\frac{V_2}{6} - \frac{V_1}{6} + \frac{V_2}{12} - \frac{60}{12} + \frac{V_2}{30} = 0$$

$$-\frac{1}{6}V_1 + V_2\left(\frac{1}{6} + \frac{1}{12} + \frac{1}{30}\right) - 5 = 0$$

$$-0.167V_1 + 0.283V_2 = 5 \longrightarrow ②$$

* By solving eq ① and ②

$V_1 = 181.71$ Volts.

$V_2 = 124.89$ Volts.

* $V_x$ = Voltage across 3Ω = Node Voltage $V_1 = 181.7$ Volts.
* $V_y$ = Voltage across 12Ω = $V_2 - 60 = 64.89$ Volts.

* Power consumed by $6\Omega$ resistance $P_{6\Omega}$

$$P_{6\Omega} = \frac{(V_2 - V_1)^2}{6}$$

Power (P) = V2.

Current $I = \frac{V}{R}$

$P = \frac{V^2}{R}$

$$= \frac{(-56.82)^2}{6}$$

$P_{6\Omega} = 538.085$ Watts.

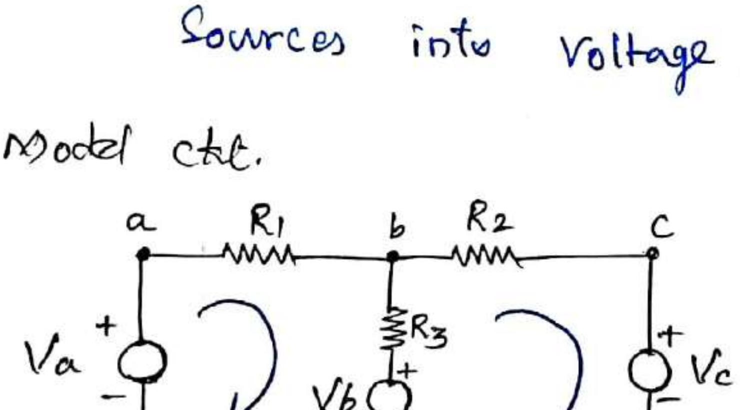
# MESH ANALYSIS.

⇒ Steps to be followed.

* Identify independent circuit meshes.
* Assign circulating current to each branch mesh.
* Usually clockwise direction is assigned to all mesh current.
* Apply KVL to all meshes and derive the equations for each mesh.
* By solving all the equations, can find the all mesh currents.

Note : If any current Sourse is present in the given circuit, have to Convert all the Current Sources into Voltage Source.

Model ckt.




* KVL equations,
  - ↳ Mesh - 1.

    $$R_1 i_1 + R_3(i_1 - i_2) = (V_a - V_b) \longrightarrow ①$$

  - ↳ Mesh - 2.

    $$R_3(i_2 - i_1) + R_2 i_2 = (V_b - V_c) \longrightarrow ②$$

* After simplified eq ①, ②

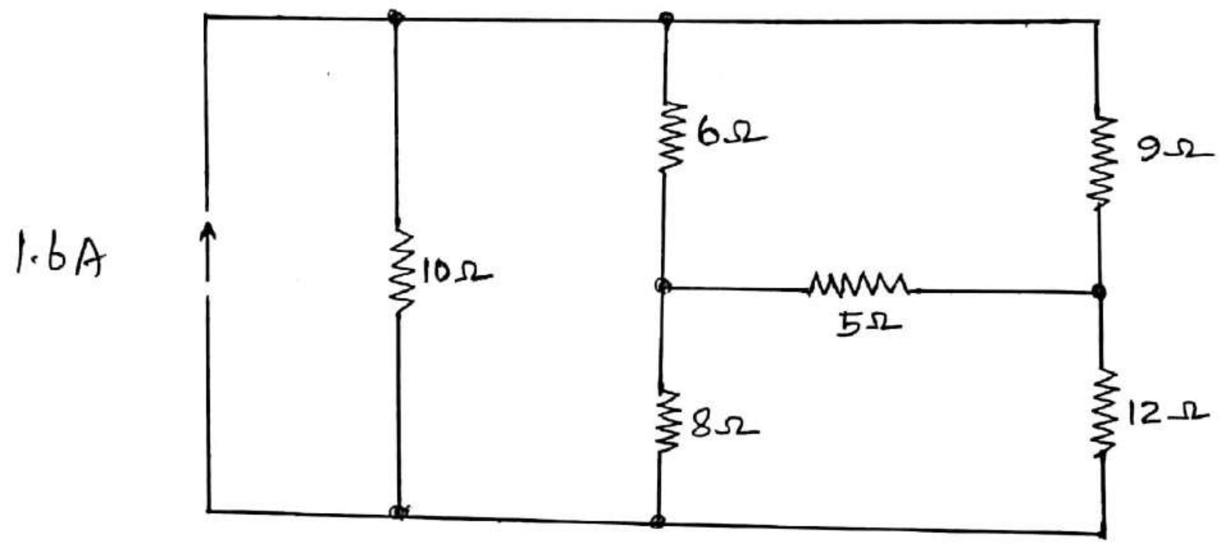
$$① \rightarrow (R_1 + R_3) i_1 - R_2 i_2 = (V_a - V_b) \longrightarrow ③$$

$$② \rightarrow -R_3 i_1 + (R_2 + R_3) i_2 = (V_b - V_c) \longrightarrow ④$$

* By solving eq. ③ & ④ using matrix method can find the mesh currents.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (V_a - V_b) \\ (V_b - V_c) \end{bmatrix}$$

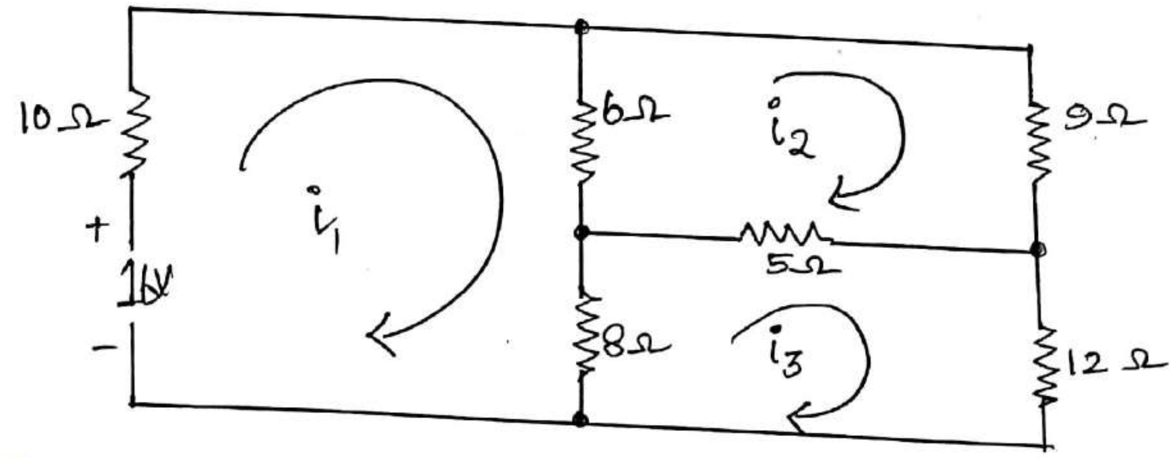
Ex. No: 1. Analyse the given circuit by mesh method, From the results, calculate the Current in the $5\Omega$ resistance.




Soln

* Convert the current source 1.6A into Voltage source.

New Voltage Source = $1.6A * 10\Omega$ = 16 Volts.




* Apply KVL in mesh - 1.

$$10i_1 + (i_1 - i_2)6 + (i_1 - i_3)8 = 16$$

$$10i_1 + 6i_1 - 6i_2 + 8i_1 - 8i_3 = 16$$

$$24i_1 - 6i_2 - 8i_3 = 16 \longrightarrow ①$$

* Apply KVL in mesh - 2.

$$6(i_2 - i_1) + 9i_2 + 5(i_2 - i_3) = 0$$

$$6i_2 - 6i_1 + 9i_2 + 5i_2 - 5i_3 = 0$$

$$-6i_1 + 20i_2 - 5i_3 = 0 \longrightarrow (2)$$

* Apply KVL in mesh - 3.

$$8(i_3 - i_1) + 5(i_3 - i_2) + 12i_3 = 0$$

$$8i_3 - 8i_1 + 5i_3 - 5i_2 + 12i_3 = 0$$

$$-8i_1 - 5i_2 + 25i_3 = 0 \longrightarrow (3)$$

* Solve eq. (1), (2) and (3) by matrix method. [Maxwell's loop current method]

$$\begin{bmatrix} 24 & -6 & -8 \\ -6 & 20 & -5 \\ -8 & -5 & 25 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$$

$$R \quad I = V$$

$$\Delta = \begin{bmatrix} 24 & -6 & -8 \\ -6 & 20 & -5 \\ -8 & -5 & 25 \end{bmatrix} = 8740$$

$$\Delta_1 = \begin{bmatrix} 16 & -6 & -8 \\ 0 & 20 & -5 \\ 0 & -5 & 25 \end{bmatrix} = 7600$$

$$\Delta_2 = \begin{bmatrix} 24 & 16 & -8 \\ -6 & 0 & -5 \\ -8 & 0 & 25 \end{bmatrix} = 3040$$

$$\Delta_3 = \begin{bmatrix} 24 & -6 & 16 \\ -6 & 20 & 0 \\ -8 & -5 & 0 \end{bmatrix} = 3040$$

* Now mesh-1 current $i_1 = \frac{\Delta_1}{\Delta} = \frac{7600}{8740} = 0.869\,A$

mesh-2 Current $i_2 = \frac{\Delta_2}{\Delta} = \frac{3040}{8740} = 0.348\,A$

mesh-3 Current $i_3 = \frac{\Delta_3}{\Delta} = \frac{3040}{8740} = 0.348\,A.$

* Now Current through $5\Omega$ Resistor $= i_2 - i_3$

$= 0.348 - 0.348$

$= 0\,A.$

Note: In a balanced bridge, suppose a resistor with any value is connected across it, the concern new resistor does not carry any current.

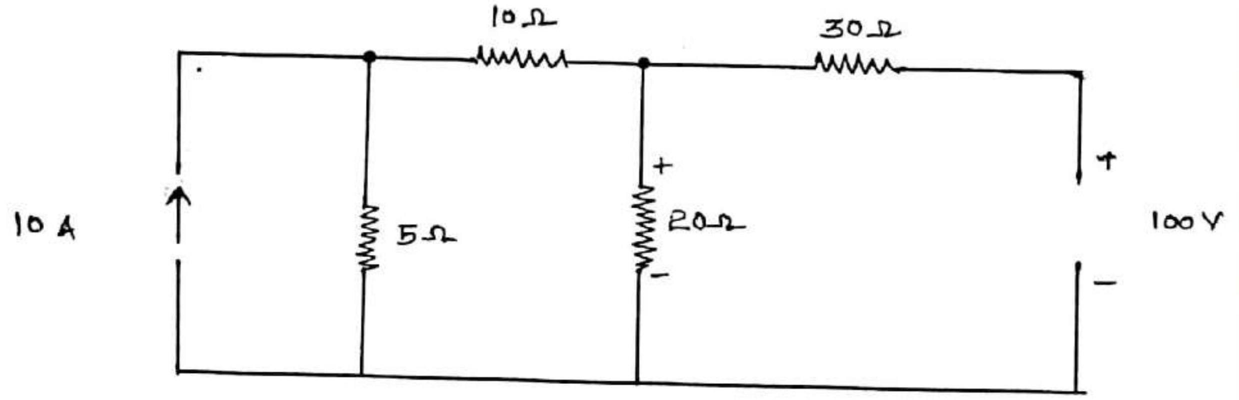
* In the given circuit Resistors $6\Omega$, $8\Omega$, $9\Omega$ and $12\Omega$ are connected together to form a balanced bridge.

$$\frac{6\Omega}{8\Omega} = \frac{9\Omega}{12\Omega} = \frac{2}{3} = \text{equal bridge arm ratio.}$$

↳ So it is a balanced bridge.

=x=

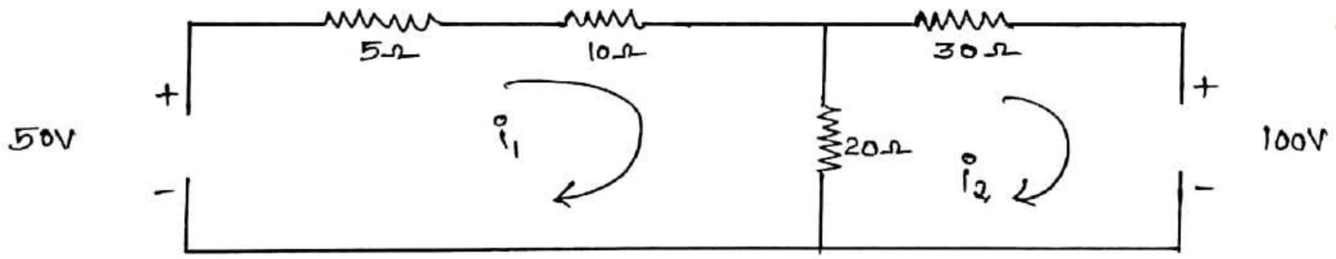
Ex. No : 2. For the given circuit determine the voltage across 20 Ω Resistance using mesh analysis.




Sol

* Convert 10A Current source into voltage source.

$$V = IR = 10 * 5 = 50 \text{ Volts.}$$




* Apply KVL in mesh - I.

$$5i_1 + 10i_1 + 20(i_1 - i_2) = 50$$

$$15i_1 + 20i_1 - 20i_2 = 50$$

$$35i_1 - 20i_2 = 50 \longrightarrow ①$$

* Apply KVL in mesh - II.

$$20(i_2 - i_1) + 30i_2 = -100$$

$$20i_2 - 20i_1 + 30i_2 = -100$$

$$-20i_1 + 50i_2 = -100 \longrightarrow ②$$

* By using Maxwell's current loop method.

$$\begin{bmatrix} 35 & -20 \\ -20 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

* Determinant $\Delta = \begin{bmatrix} 35 & -20 \\ -20 & 50 \end{bmatrix} = 1350$

$$\Delta_1 = \begin{bmatrix} 50 & -20 \\ -100 & 50 \end{bmatrix} = 500$$

$$\Delta_2 = \begin{bmatrix} 35 & 50 \\ -20 & -100 \end{bmatrix} = -2500$$

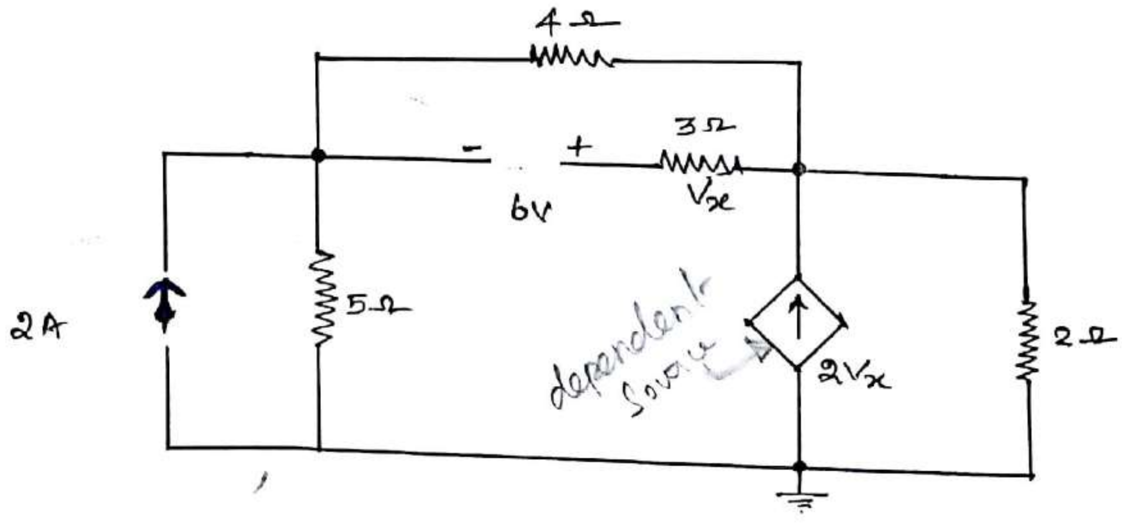
* Now the mesh current $i_1 = \frac{\Delta_1}{\Delta} = \frac{500}{1350} = 0.37\,A$

mesh current $i_2 = \frac{\Delta_2}{\Delta} = \frac{-2500}{1350} = -1.85\,A.$

* Now the voltage across $20\,\Omega$ Resistor $= |20(i_2 - i_1)|$

$$V_{20\Omega} = 44.4 \text{ Volts.}$$

Ex No: 3. For the given circuit with a dependent source, Find the value of $V_x$ using mesh analysis.



| Nodal Analysis | Mesh Analysis. |
|---|---|
| * Final answer will be Voltage. | * Final answer will be Current. |
| * More no. of nodes leads to critical in calculation | * More no. of meshes can solve easy manner by using Maxwell's current loop method. |

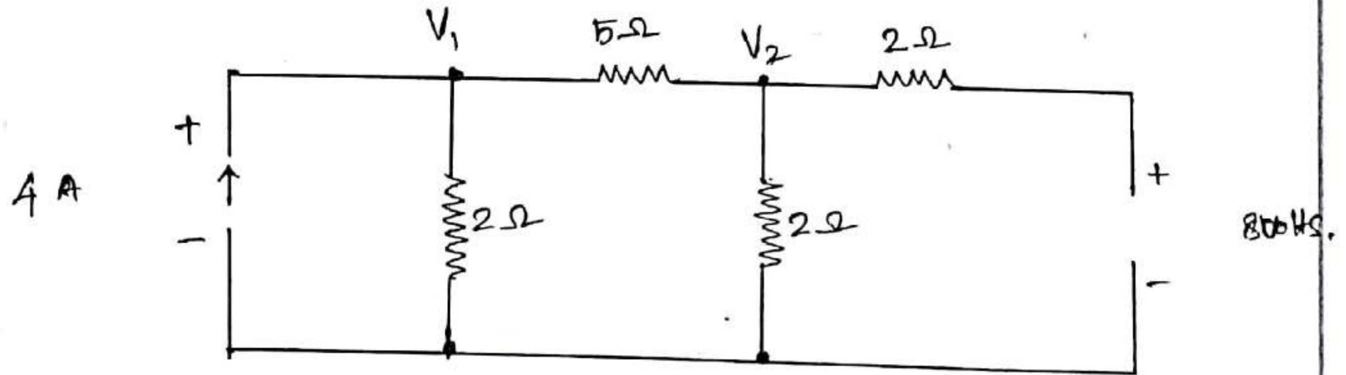
## NETWORK THEOREMS.

### 1. Superposition theorem

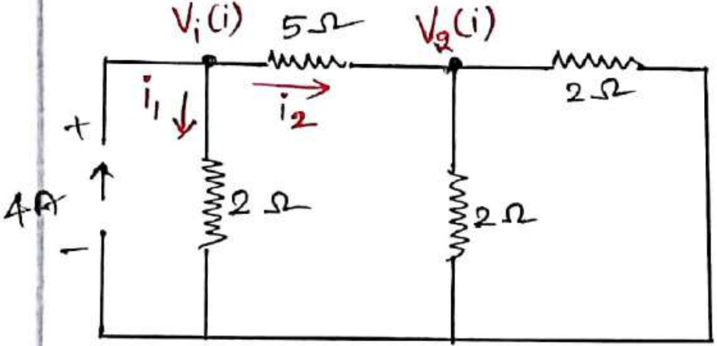
The total current in any part of a linear circuit equals the algebraic sum of the current produced by each source seperately. To evaluate the seperate currents to be combined, replace all other voltage sources by shot circuits and all other current sources by open circuits.

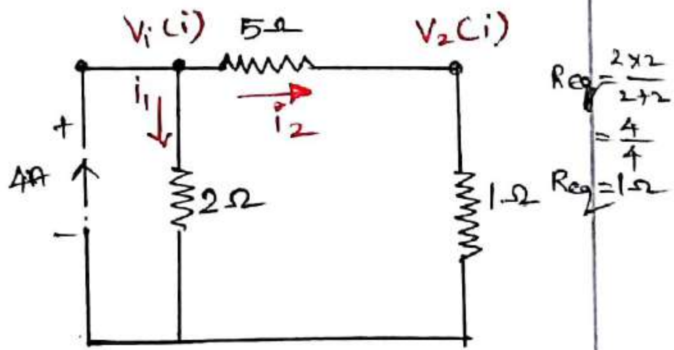
Ex.NO: 1

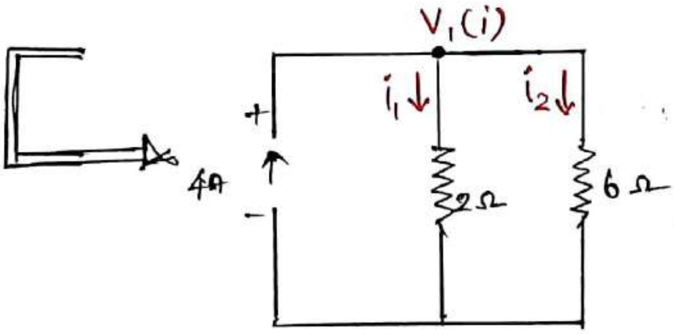
Find the node voltages $V_1$ and $V_2$ using superposition theorem.



[*] only current source is applied and voltage source is short circuited.



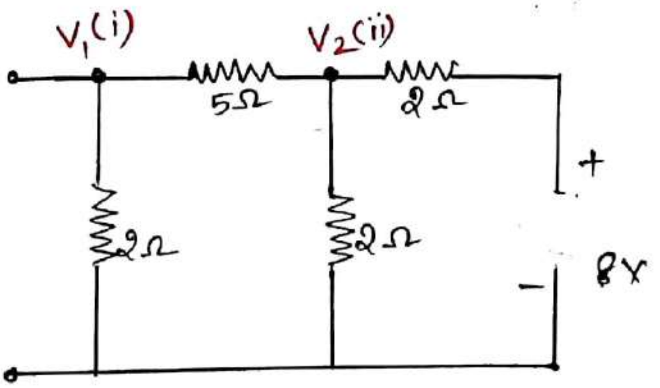



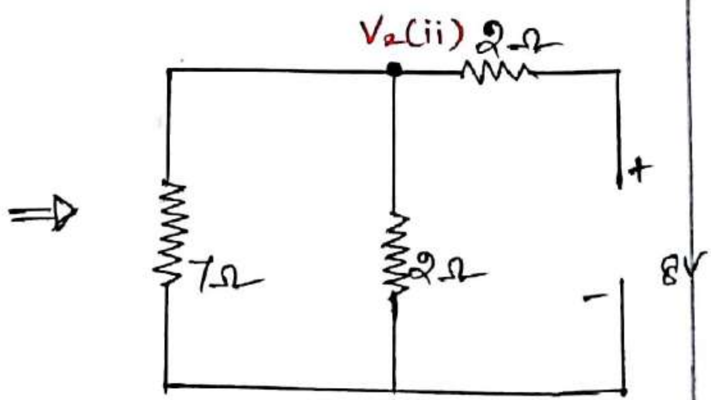

* from the above diagram

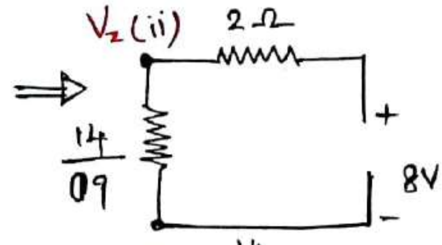
$$i_1 = 4 * \frac{6}{6+2} = 3A. \qquad V_1(i) = i_1 * 2\Omega = 6 \text{ Volts.}$$

$$i_2 = 4 * \frac{2}{6+2} = 1A \qquad V_2(i) = i_2 * 6\Omega = 1 \text{ Volts.}$$

[*] Now only voltage source is applied and the current source is open circuited.



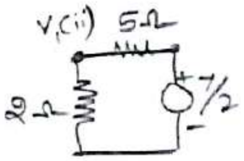



$\frac{7*2}{7+2} = R_{eq} = \frac{14}{9}$

* from the above figures,

$$V_2(ii) = 8V * \frac{(14/9)}{2+(14/9)} = \frac{7}{2} \text{ Volts.}$$

$$V_1(ii) = \frac{7}{2} * \frac{2}{2+5} = 1 \text{ Volts.}$$




* Now using Superposition theorem,

$$V_1 = V_1(i) + V_1(ii) = 6+1 = 7 \text{ Volts.}$$

$$V_2 = V_2(i) + V_2(ii) = 1+\frac{7}{2} = \frac{9}{2} \text{ Volts.}$$

Ans

Node Voltages

$V_1 = 7$ Volts.

$V_2 = 9/2$ Volts.

## 2. Thevenin and Norton theorems.

### Thevenin's theorem.

Any combination of resistances with two terminals can be replaced by a single voltage source and single series resistor. (in series)

### Norter's theorem.

Any combination of resistances with two terminals can be replaced by a single current source and single parallel resistor.

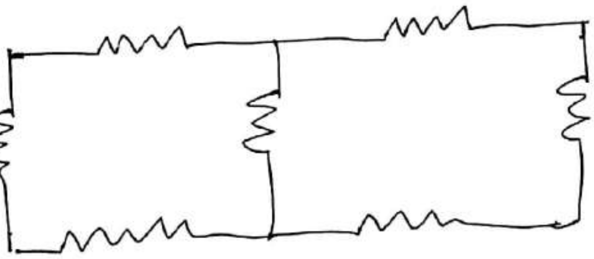
Purpose ⇒ reduce ckt complexity & making simple calculation.

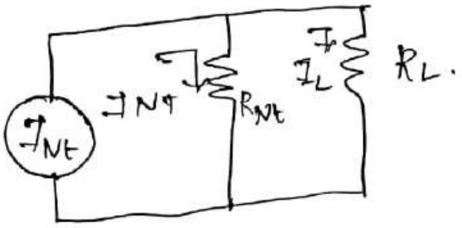
## Nortorn's theorem:

* The entire linear circuit is replaced by only one current source with parallel resistance.

* this nortorn's theorem is only applicable for linear circuits.

* By using this nortorn's theorem only one branch current can be found.






$$I_L = \frac{I_{NT} * R_{NT}}{R_{NT} + R_L}.$$

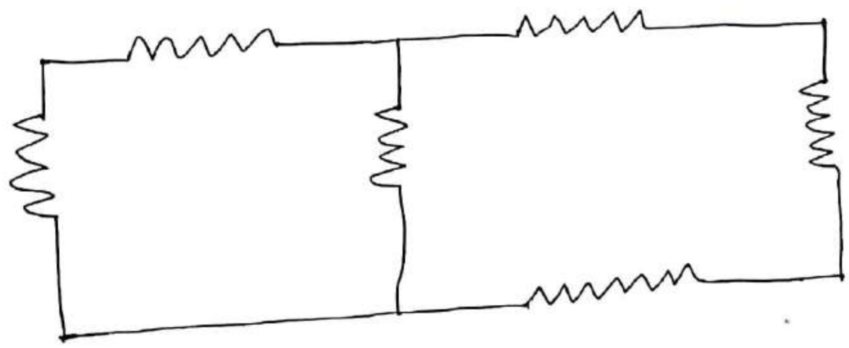
Thevenin's theorem:

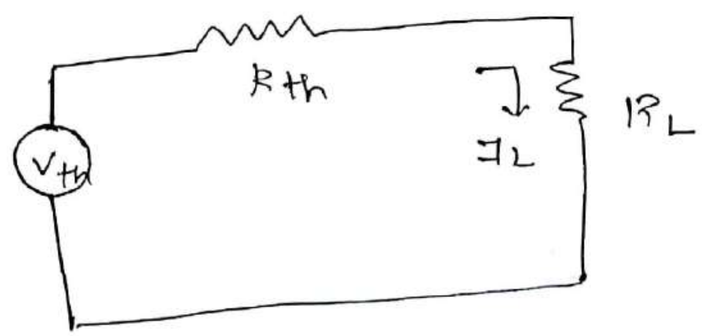
* The entire linear circuit is replaced by only one voltage source with series resistance.

* This thevenin's theorem only applicable for linear circuits.

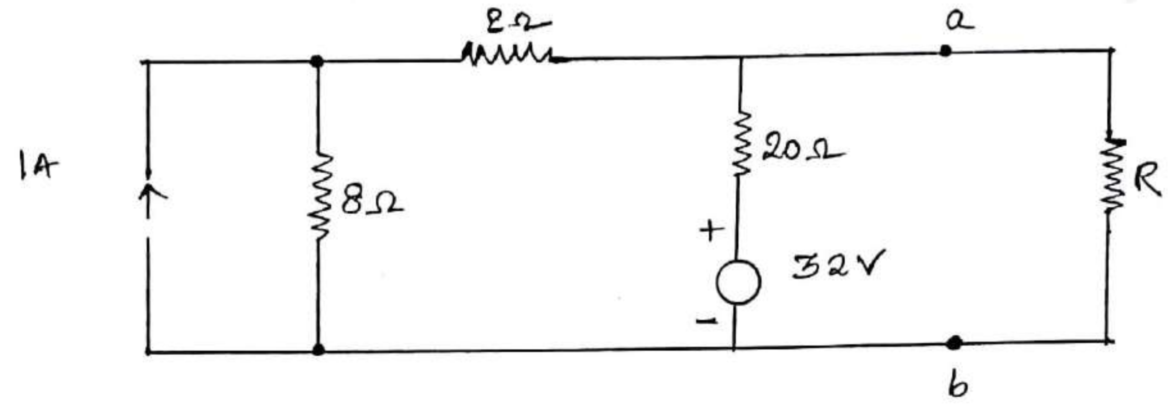
* By using this thevenin's theorem only one branch current can be found.

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$





Ex. No:1. Find the thevenin and Norton equivalents of the given ckt as seen at terminal a,b.




Sol:

→ Thevenin's equivalent.

* Remove load Resistance 'R' by making open ckt at ab.
* Replace 1A current source and 8Ω Parallel resist by it's equivalent voltage source.

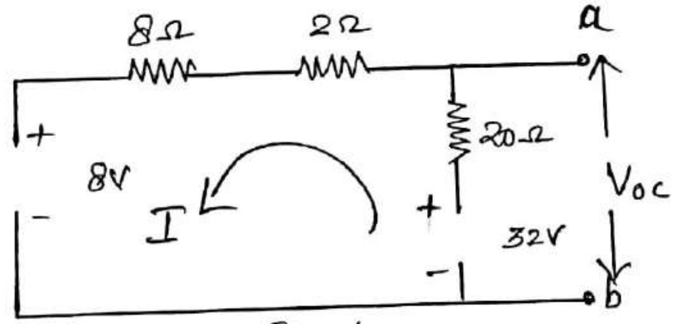



Fig -1.

* Voltage source

  8V < 32V

* When in a single loop more than one voltage sources present the loop current direction depends the bigger voltage :-

* From the above fig,

$$\text{current } I = \frac{32V - 8V}{20 + 2 + 8}$$

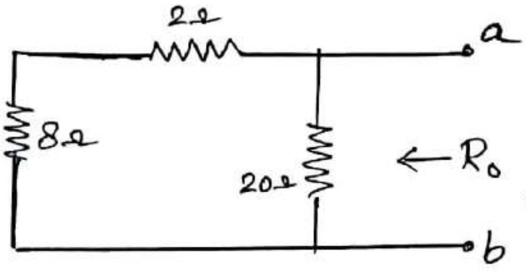
$$I = 0.8 \text{ Amp.}$$

Now the open circuit Voltage $V_{oc}$

$$V_{oc} = 32V - (20 * 0.8)$$

$$V_{oc} = 16 \text{ Volts.}$$

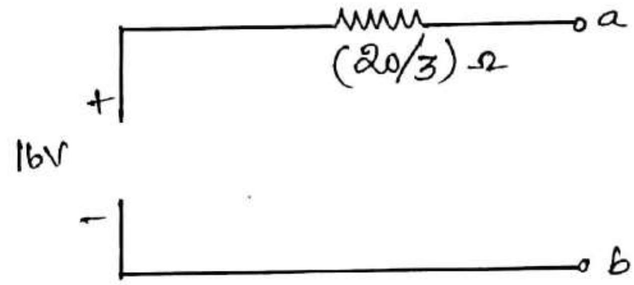
* Now in the given fig.1. make the current source 1A as open ckt and put short ckt for 32V. Now the new ckt will be,




* The thevenin's equivalent Resistance will be,

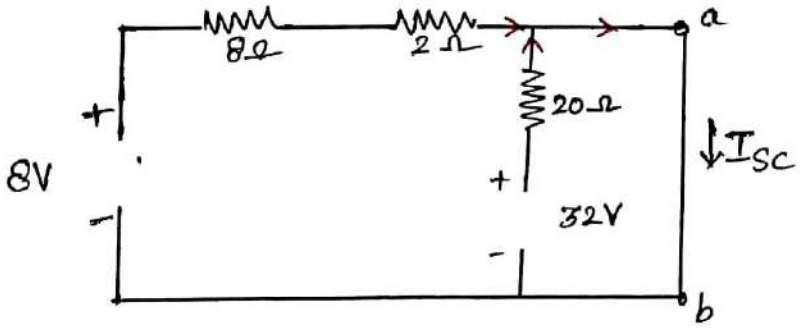
$$R_0 = \frac{20 * 10}{20 + 10} = \frac{20}{3}\ \Omega.$$

* Thevenins equivalent ckt,




→ Norton equivalent.

* Remove load Resistance and put short ckt at ab.
* Convert 1A current source into voltage source.




* From the fig. $I_{sc}$

$$I_{sc} = \frac{32V}{20\Omega} + \frac{8V}{10\Omega}$$

$$I_{sc} = 2.4\ \text{Amp.}$$

* As calculated in thevenin equivalent $R_0 = \frac{20}{3}\ \Omega$.
* Now Norton equivalent ckt,

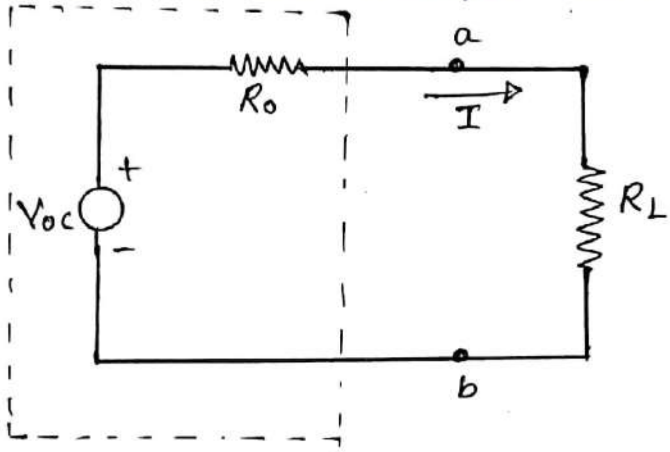

Check.

$V_{oc} = 2.4 * \frac{20}{3}$

$V_{oc} = 16$ Volts. Same as in thevenin's equivalent.

# Maximum Power transfer theorem.

The power delevered to load is maximum when load Resistance ($R_L$) equals the thevenin's resistance ($R_{th}$) of the source is known as maximum power transfer theorem.




* To deliver maximum power to load,

$$R_o = R_L$$

where,

$$R_o = R_{th}.$$

* Network current (I)

$$I = \frac{V_{oc}}{R_o + R_L}$$

PROOF.

* Expression for load power $P_L = I^2 R_L$

$$P_L = \left(\frac{V_{oc}}{R_o + R_L}\right)^2 R_L$$

* To find maximum power delevered to load

$$\frac{dP_L}{dR_L} = 0$$

↳ which gives the result $R_L = R_o$.

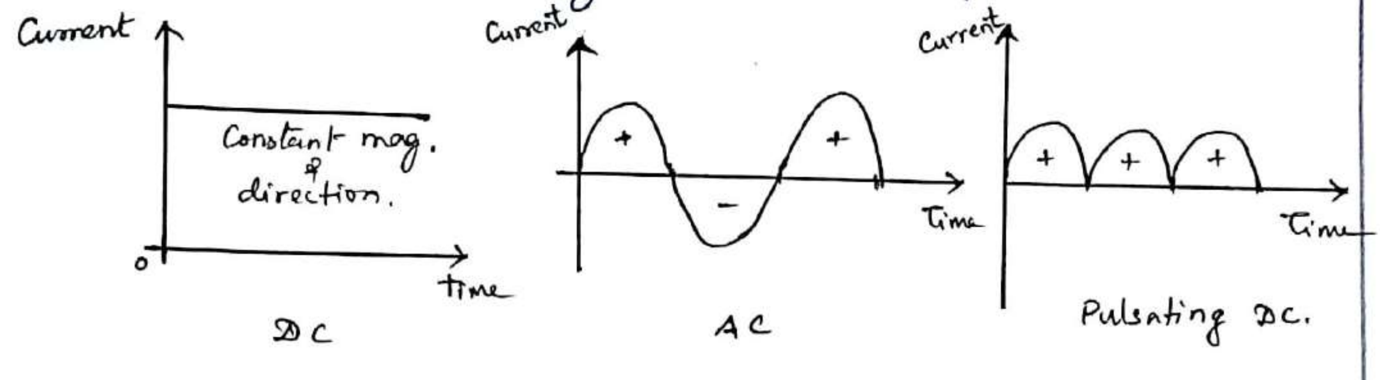
* Power output by the source $P_S = V_{oc} I = V_{oc} \frac{V_{oc}}{R_o + R_L} = \frac{V_{oc}^2}{2R_L}$
* Now load power $P_L = \left(\frac{V_{oc}}{2R_L}\right)^2 R = \frac{V_{oc}^2}{4R_L}$
* Now the efficiency of Power transfer $(\% \eta) = \frac{\text{load Power}}{P_S} \times 100$

$$\% \eta = \frac{1}{2} \times 100 = 50\%.$$

# AC CIRCUITS.

## Introduction.

* AC ⇒ changes periodically both in magnitude and direction
* DC → Constant magnitude with respect time.




* AC ⇒ Step up and step down operations can be done by Transformers.
* HVAC Transmission ⇒ economical & efficient.
* Cost of alternators is low, upto 11KV it is possible to generate.
* AC motors are simple in construction and less maintanance would be brought.
* Easy to convert DC wherever it is required.

⇒ Instantaneous Value.

The value of alternating quantity at a particular instant is known as "Instantaneous value".

$$I = i_m \sin(\omega t) \qquad e = E_m \sin(\omega t).$$

→ Waveform.

The graph of alternating quantity instantaneous values against time is called as Waveform.

⇒ Cycle.

A set of +ve and -ve instantaneous values of an alternating quantity is known as cycle.

⇒ Time Period.

It is a time to complete one cycle of an alternating quantity, denoted by 'T'. $T = 1/f$ || f = freq.

⇒ Frequency

No. of cycles completed by an alternating quantity per one second is called freq. denoted by f.

$$f = \frac{1}{T} \text{ in Hz.}$$

⇒ Amplitude.

The maximum value attained by an alternating quantity during +ve and -ve half cycles is called amplitude. Also called as "Peak value" or "Maximum Value".

⇒ Angular frequency. [ω]

* It is the frequency expressed in radians/second.
* $\omega = 2\pi f$ (or) $(2\pi/T)$ in radian/sec.

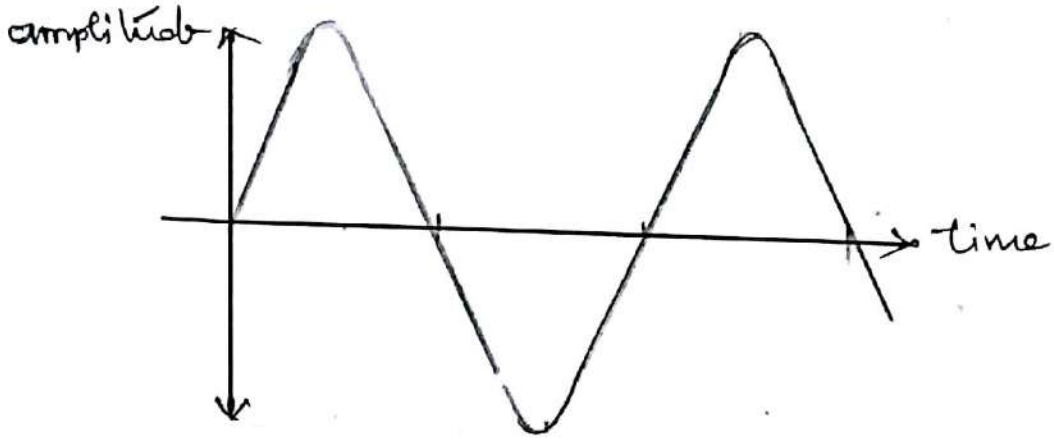
⇒ Peak to Peak Value.

The value of Alternating quantity from (+ve) peak to (-ve) peak is called as peak to peak.

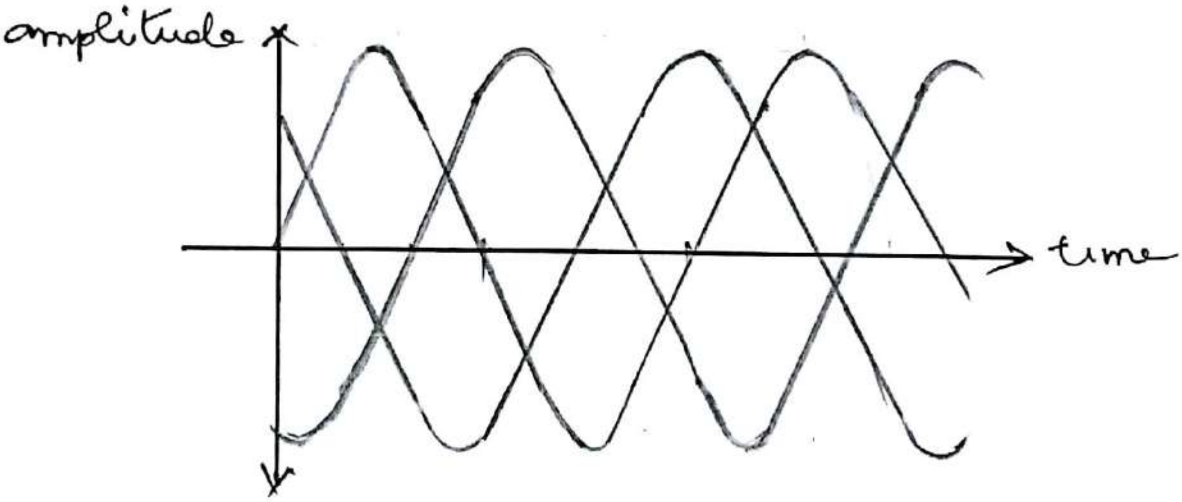
↳ Peak to Peak = 2 * Amplitude.

Wave forms.

Single phase waveforms.




Three phase AC Waveforms.

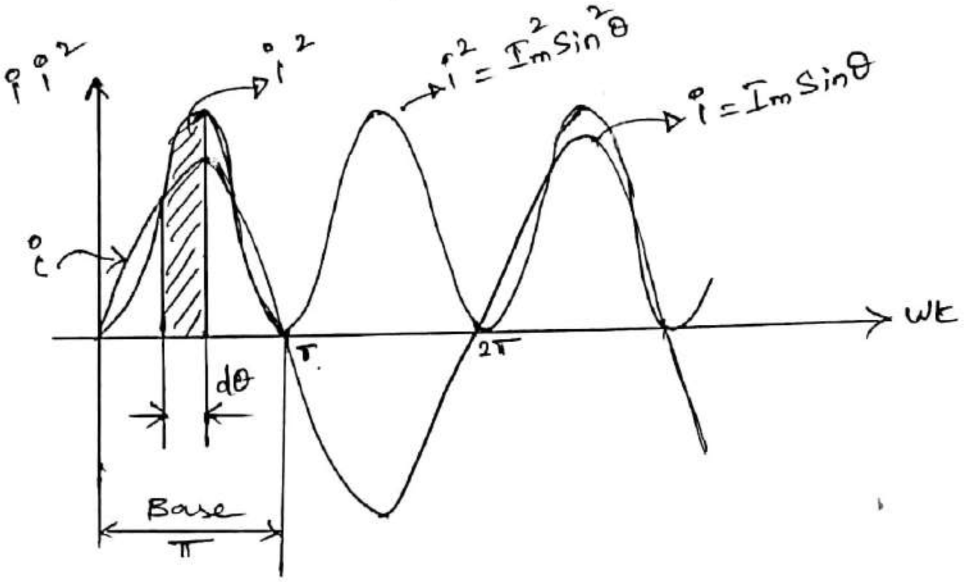



RMS value. (or) Effective value.

* The RMS value is the effective value of a varying voltage (or) current.
* It is the equivalent steady DC value which gives the same effect.

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{E_m}{\sqrt{2}}$$

## Analytical Method for RMS value.




* We know the expression for instantaneous value of Current $i = I_m \sin\theta$

Square of Current $i^2 = I_m^2 \sin^2\theta$

* Now the Root Mean Square (RMS) value

$$\text{RMS value} = \sqrt{\text{mean (or) average value}}$$

* Mean (or) average value of Current (or) Voltage

$$\text{Average value} = \frac{\text{Area of Curve over one half cycle of squared waveform.}}{\text{Length of cycle.}}$$

→ from the fig.,

* Area of square Curve over one half cycle $\Big\} = \int_0^{\pi} i^2 d\theta$

* lenth of the base is $\pi$



Prepared by Dr.N.Balamurugan

* Now the average value of square of the current over one half cycle is,

$$= \frac{\int_0^{\pi} i^2 \, d\theta}{\pi}$$

$$= \frac{1}{\pi} \int_0^{\pi} i^2 \, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi} \int_0^{\pi} \left[ \frac{1 - \cos 2\theta}{2} \right] d\theta$$

$$= \frac{I_m^2}{2\pi} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi} \left[ (\pi - 0) - (0 - 0) \right]$$

$$= \frac{I_m^2}{2\pi} (\pi)$$

$$= \frac{I_m^2}{2}.$$

* RMS value of current $I_{rms}$.

$$I_{rms} = \sqrt{\left(\frac{I_m^2}{2}\right)} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m.$$

* Simillarly value of voltage $V_{rms}$

$$V_{rms} = 0.707 \, V_m.$$

=x=

Average value.

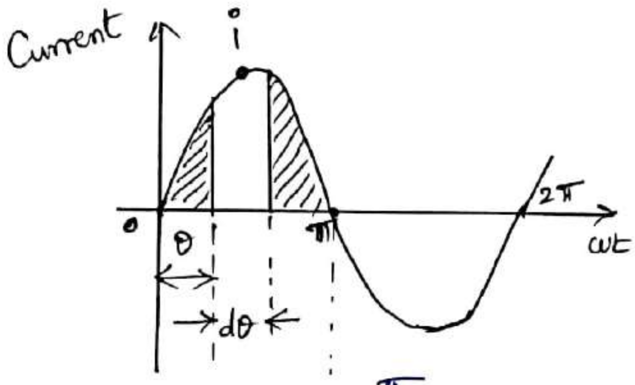
* The average value of a whole sinusoidal waveform over one complete cycle is zero as the two halves cancel each other out, so the average value is taken over one half cycle.

* The average value of sine wave of voltage (or) current is 0.637 times the peak value.

$$I_{avg} = 0.637\, I_{peak} = 0.637\, I_m$$

$$V_{avg} = 0.637\, V_{peak} = 0.637\, V_m.$$

Analytical Method for average value.




* Instantaneous current $i = I_m \sin \omega t$.

* $I_{av} = \dfrac{\text{Area Under Curve for Half cycle}}{\text{Length of base over half cycle}}$.

$$I_{av} = \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}(-\cos\theta)_0^{\pi}$$

$$= \frac{I_m}{\pi}(2) = \frac{2I_m}{\pi}$$

$$I_{av} = 0.637\, I_m$$

|||y $V_{av} = 0.637\, V_m.$

Form Factor ($K_f$)

$$K_f = \frac{\text{RMS value}}{\text{Avg. value}}$$

$$K_f = \frac{0.707 I_m}{0.637 I_m}$$

$K_f = 1.11$ for sinusoidal wave.

Crest (or) Peak factor. ($K_p$)

$$K_p = \frac{\text{Maximum Value}}{\text{r.m.s value.}}$$

$$K_p = \frac{I_m}{0.707 I_m} = 1.414$$

$K_p = 1.414$ for Sinusoidal wave.

Power and power factor in AC circuit.

→ DC circuits

* Power (P) = V * I.
* Current (I) always inphase with Voltage.
* No phase difference between V and I.

→ AC circuits.

* Power (P) = V I Cos $\phi$.
* V and I may have phase difference between them. (phase angle = $\phi$)
* Power consumption by AC ckt is depends "$\phi$".

Power factor

* Cosine of the phase angle ($\phi$) is called as power factor.

$$\text{power factor} = \cos\phi$$

* Power depends on elements of the circuit. [elements such as R, L & C]

* Leading P.F $\Rightarrow$ I leads V by angle $\phi$.
Lagging P.F $\Rightarrow$ I lags V by angle $\phi$.

$$\text{Power factor} = \frac{\text{True Power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI} = \cos\phi.$$

$$\text{Power factor} = \frac{R}{Z}$$

↳ Apparent power. (S)

* Product of r.m.s value of voltage and r.m.s value of current.

$$S = V * I \quad \text{in VA.}$$

↳ Real (or) True power (P)

* Product of applied voltage and the active component of current.

$$P = VI\cos\phi \quad \text{in watts}$$

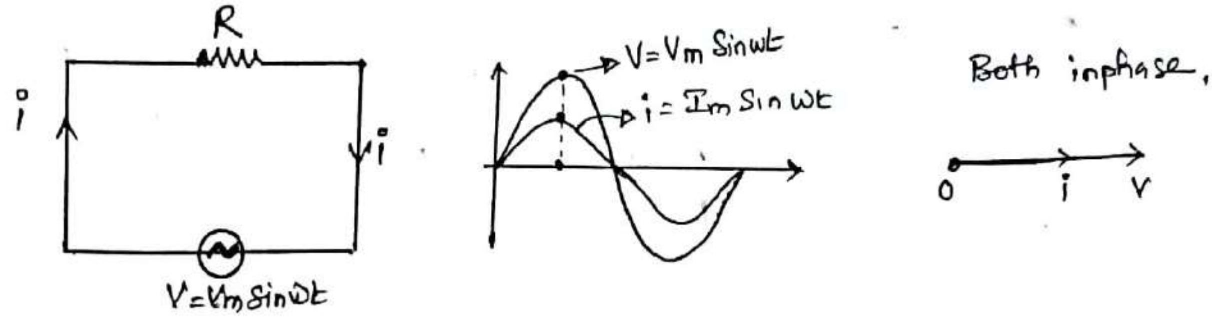
↳ Reactive power (Q)

* Product of applied voltage and the reactive component of current.

$$Q = VI\sin\phi \quad \text{in VAR.}$$

## AC through pure Resistance.

* A pure resistance 'R' connected across voltage.




* From ohm's law $i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$

$$I_m = \frac{V_m}{R} \quad \text{and } \phi = 0^\circ.$$

* Intantaneous Power (P) $= V * i = V_m \sin\omega t * I_m \sin\omega t.$

$$= V_m I_m \sin^2\omega t.$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

constant power, fluctuating power.

$$P_{ins.} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t.$$

* Average power over one half cycle.

↳ over one half cycle cosine component of double frequency is equal to zero.

$$P_{av} = \frac{V_m I_m}{2} - 0$$

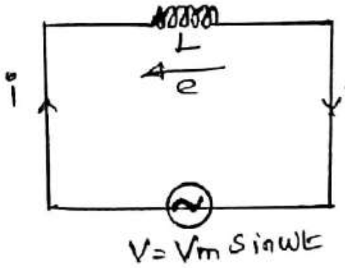
$$= \frac{V_m I_m}{\sqrt{2}\,\sqrt{2}}$$

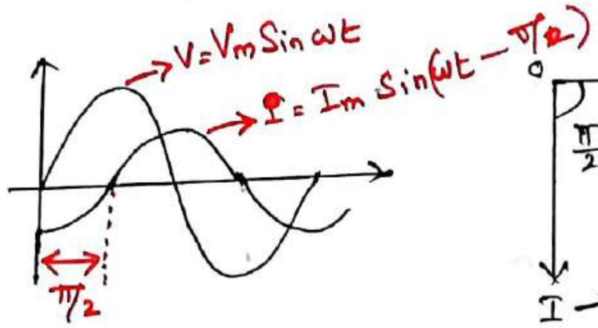
$$P_{av} = V_{rms} * I_{rms} \quad \text{in watts.}$$

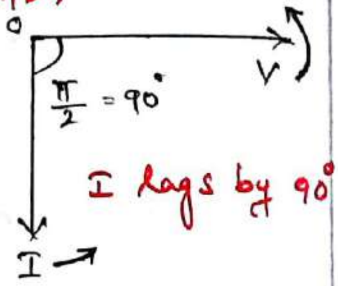
$$P_{av} = VI = I^2 R \quad \text{in Watts.}$$

# AC through pure inductance.

* A pure inductance connected across voltage.




* Pure inductance has zero ohmic resistance.
* When alternating current flows through the 'L', it sets up an alternating mag. field arround the 'L' and due to self inductance an emf develops in 'L', this emf opposes the supply voltage.

$$\text{Self induced emf in coil} = -L\,\frac{di}{dt}$$

* At all instants, applied voltage (V) is equal and opposite to the self induced emf (e)

$$V = (-e) = -(-L\,di/dt)$$

$$V_m \sin\omega t = L\,(di/dt)$$

$$di = \frac{V_m}{L}\sin \omega t\,dt.$$

$$i = \int di = \int \frac{V_m}{L}\sin(\omega t)\,dt$$

$$= \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$= \frac{-V_m}{\omega L}\left(\sin\left(\pi/2 - \omega t\right)\right)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/2\right)$$

$$i = I_m \sin(\omega t - \pi/2)$$

where, $I_m = \dfrac{V_m}{\omega L} = \dfrac{V_m}{X_L}$ and $X_L = 1/\omega L = 1/2\pi f L \ \Omega$.

* Instantaneous power $(P) = v \times i$

$$= V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$$

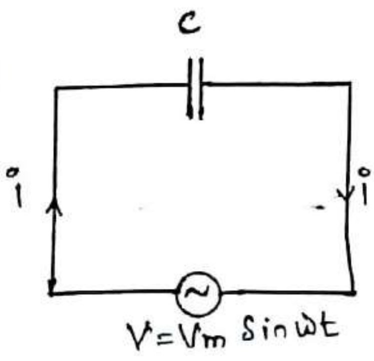
$$= -V_m I_m \sin(\omega t) \cos(\omega t)$$

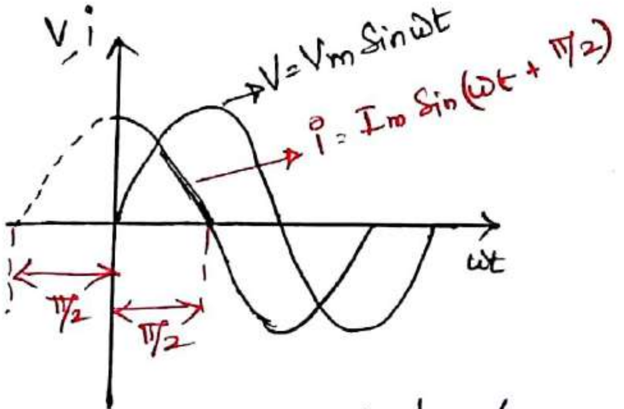
$$P = -\frac{V_m I_m}{2} \sin(2\omega t)$$

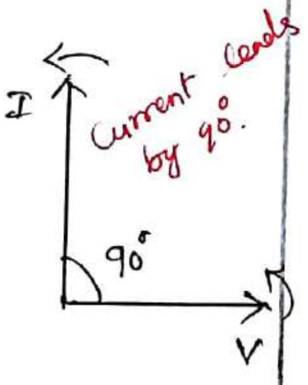
* Average Power $(P_{av}) = \displaystyle\int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$

(X) The area of +ve loop and (−ve) loop are exactly same and hence, the average power consumption is zero.

## AC through pure Capacitance.




* current '$i$' charges the capacitor '$C$'.
* Now the intantaneous charge '$q$' on the plates of the Capacitor is given by

$$q = CV$$

$$q = C V_m \sin \omega t.$$

* Current $(i) = dq/dt$

$$= \frac{d}{dt}(C V_m \sin \omega t)$$

$$= C V_m \frac{d}{dt}(\sin \omega t)$$

$$= C V_m \omega \cos \omega t.$$

$$= \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2)$$

Where,
↳ $\cos \omega t = \sin(\omega t + \pi/2)$
↳ $C\omega = 1/\frac{1}{\omega C}$

$$i = I_m \sin(\omega t + \pi/2)$$

* where, $I_m = \frac{V_m}{X_c}$ and $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ $\Omega$

* Instantaneous power $p = v * i$

$$P = V_m \sin(\omega t) * I_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin(\omega t) \cos(\omega t)$$

$$P = \frac{V_m I_m}{2} \sin(2\omega t)$$

* Average power $P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\, d\omega t = 0$

$$P_{av} = 0.$$

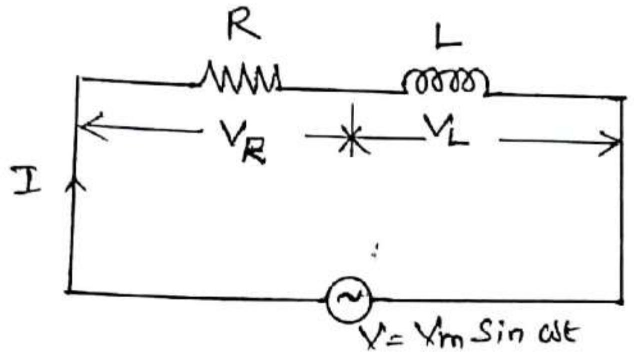
* The average value of sine curve over a complete cycle is zero.

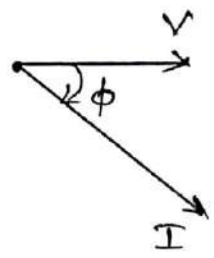
* Pure capacitance never consumes power.

* Average energy stored in a capacitor $E = \frac{1}{2} C V^2$ Joules.

=*=

## AC through R-L circuit.




I lags V by $\phi$ angle.

* RL series combination is connected across supply AC.
* There are two voltage drops 1) $V_R = I * R$ 2) $V_L = I * X_L$

  where, $X_L = 2\pi f L$, $I$ = rms value of current.

  $V_R, V_L$ = rms value of voltage drops.
* By KVL, the supply voltage $\bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}X_L$
* From the voltage triangle (V, $V_L$, $V_R$)

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$V = IZ \qquad \text{where } Z = \sqrt{R^2 + X_L^2}$$

* Here current I lags voltage V by angle $\phi$.

$$V(t) = V_m \sin \omega t$$

$$i(t) = I_m \sin(\omega t - \phi).$$

* The instantaneous power (P)

$$P = v * i$$

$$= V_m \sin(\omega t) * I_m \sin(\omega t - \phi)$$

$$= V_m I_m (\sin(\omega t) * \sin(\omega t - \phi))$$

$$P = V_m I_m \left( \frac{\cos\phi - \cos(2\omega t - \phi)}{2} \right) \text{ watts.}$$

$$P = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi) \text{ . watts.}$$

* Now the average power $P_{avg}$,

$$P_{avg} = \frac{V_m I_m}{2} \cos\phi$$

$\Rightarrow$ Average value over a complete cycle is "zero". So $\cos(2\omega t - \phi) = 0$.

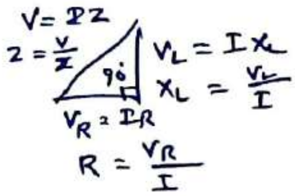
$$= \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} * \cos\phi$$

$$P_{avg} = V * I * \cos\phi \text{. watts.}$$

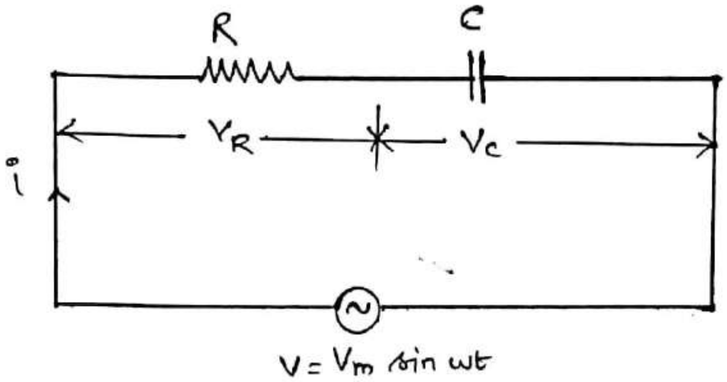
* Apparent power $(S) = VI$ in VA.
* Real power $(P) = VI\cos\phi$ in watts.
* Reactive power $(Q) = VI\sin\phi$ in VAR.
* Power factor $= \frac{\text{True Power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI} = \cos\phi = \frac{R}{Z}$

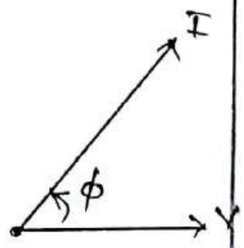
* Angle $\phi$ is (+ve) for inductive impedence.

* Power factor $\cos\phi = \frac{R}{Z}$



# AC through RC circuit.




I leads V by $\phi$.

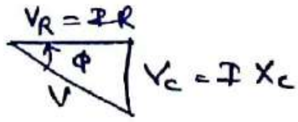
* RC Connected across supply V.
* Drop across pure R $\Rightarrow V_R = I * R$

  Drop across pure C $\Rightarrow V_C = I * X_C$

  Where, $X_C = \dfrac{1}{2\pi f C}$
* By applying KVL,

$$\bar{V} = \bar{V_R} + \bar{V_C} = \overline{IR} + \overline{I X_C}$$

* From the Voltage triangle




$$V^2 = V_R^2 + V_C^2$$

$$V = \sqrt{(IR)^2 + (I X_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$V = IZ \qquad \| \; Z = \sqrt{R^2 + X_C^2}$$

* The instantaneous Voltage and current

$$V(t) = V_m \sin \omega t$$

$$i(t) = I_m \sin(\omega t + \phi)$$

* The intantaneous power (P) $= V * i$

$$= V_m \sin(\omega t) * I_m \sin(\omega t + \phi)$$

$$= V_m I_m \left[ \frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2} \right]$$

$$P = \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$$

* Now $P_{avg} = \frac{V_m I_m}{2} \cos\phi$.

→ for a complete cycle $P_{avg} = 0$.

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi.$$

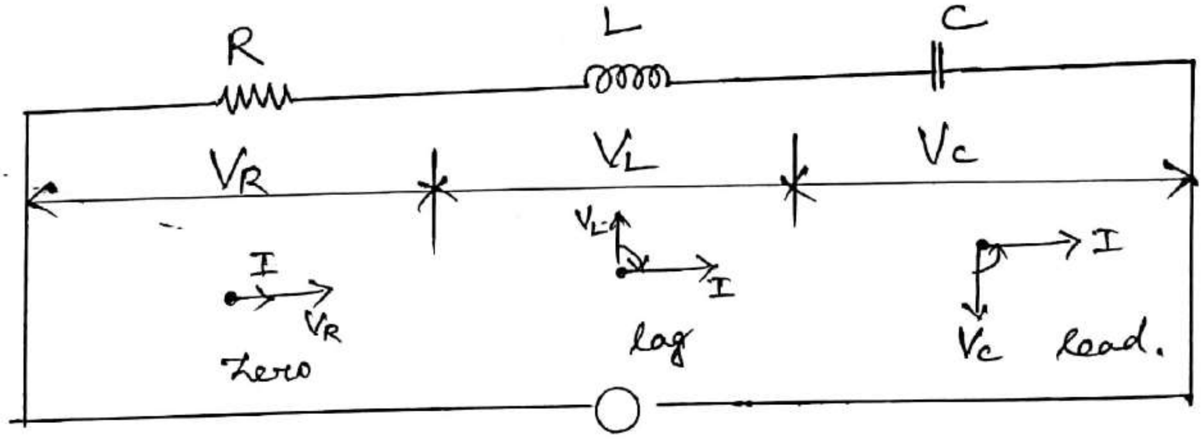
rms value

$V = \frac{V_m}{\sqrt{2}}$, $I = \frac{I_m}{\sqrt{2}}$

$$P_{avg} = V \times I \cos\phi.$$

* Apparent power $(S) = VI$ in VA.
* True (or) Avg. Power $(P) = VI \cos\phi$ in watts.
* Reactive power $(Q) = VI \sin\phi$ in VAR

## AC through RLC circuit.




* Drop across R, L and C

$V_R = IR$ — $V_R$ & $I$ → inphase.

$V_L = I X_L$ — $I$ lags $V_L$ by $90^\circ$

$V_c = I X_c$ — $I$ leads $V_c$ by $90^\circ$

* By applying KVL,

$$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_c}$$

$$V^2 = V_R^2 + V_L^2 + V_c^2$$

* $V_L$ and $V_c$ are phase opposition between themselves and depends on $X_L$ and $X_c$. Now the voltage

$$V = \sqrt{V_R^2 + (V_L \pm V_c)^2}$$

→ $X_L > X_c$ ⇒ ckt said to be inductive in nature.

$$V = \sqrt{V_R^2 + (V_L - V_c)^2}$$

$$= \sqrt{(IR)^2 + [(IX_L) - (IX_c)]^2}$$

$$= I\sqrt{R^2 + (X_L - X_c)^2}$$

$$V = IZ$$

⇒ $X_L < X_c$ ⇒ ckt said to be cappacitance in nature

$$V = I\sqrt{R^2 + (X_c - X_L)^2}$$

$$V = IZ.$$

→ $X_L = X_c$ ⇒ Will cancel each other and resultant will be zero.

$$V = I\sqrt{R^2 + (0)^2}$$

$$V = IR. = IZ. \qquad \| \ Z = R$$

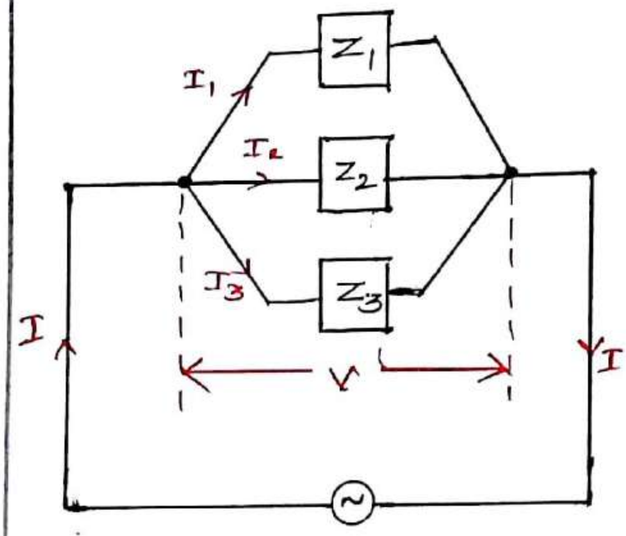
* Average power consumed by the circuit

$$P_{avg} = (P_{avg} \text{ by } R) + (P_{avg} \text{ by } L) + (P_{avg} \text{ by } C)$$

* But pure L and C doesn't consume any power.

$$P_{avg} = (P_{avg} \text{ by } R) = I^2 R = I\,\underline{V_R}. = I.V\cos\phi.$$

AC Parallel ckt.




* from KCL,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

* Two impedence in parallel,

$$\bar{I}_1 = \bar{I}_T \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$I_2 = \bar{I}_T \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}$$

→ Admittance (Y).

$$Y = G \pm B. = \frac{1}{Z}$$

where,

$$G = \text{Conductance} = \frac{R}{Z^2}$$

$$B = \text{Susceptance} = \frac{X}{Z^2}$$

↳ B = −ve if inductive.

B = +Ve if Positive.

* Admittance in parallel circuit,

$$Y_{eq} = Y_1 + Y_2 + Y_3$$

* Power factor $\cos\phi = \frac{G_{eq}}{Y_{eq}}$

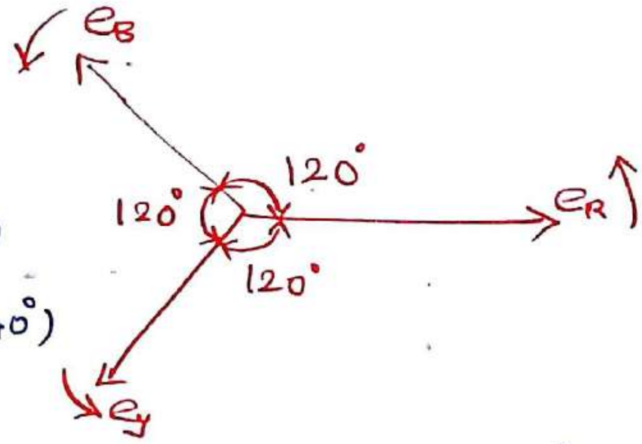
# Three phase System.

* Three phase Voltages

$$e_R = E_m \sin(\omega t)$$

$$e_Y = E_m \sin(\omega t - 120^\circ)$$

$$e_B = E_m \sin(\omega t - 240^\circ)$$




* Vector sum of three voltages at any instant is "Zero."

$$\bar{e}_R + \bar{e}_y + \bar{e}_B = E_m \sin(\omega t) + E_m \sin(\omega t - 120^\circ) + E_m \sin(\omega t - 240^\circ)$$

$$= E_m [\sin(\omega t) + \sin(\omega t - 120^\circ) + \sin(\omega t + 120^\circ)]$$

$$= E_m [\sin(\omega t) + (\sin \omega t \cos 120^\circ - \cos \omega t \sin 120^\circ) + (\sin \omega t \cos 120^\circ + \sin \omega t \cos 120^\circ)$$

$\sin(A+B) =$
$\sin(A-B) =$

$$= E_m [\sin(\omega t) + 2 \sin \omega t \cos 120^\circ]$$

$$= E_m [\sin(\omega t) + 2 \sin \omega t (-\tfrac{1}{2})]$$

$$= E_m [\sin(\omega t) - \sin(\omega t)]$$

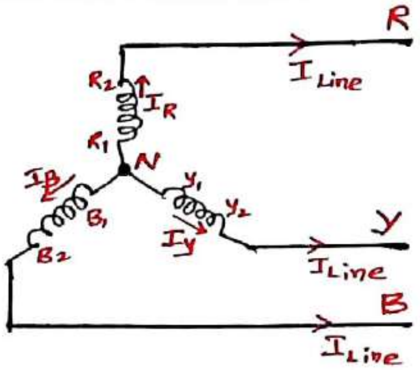
$$= E_m [0]$$

$$\bar{e}_R + \bar{e}_y + \bar{e}_B = 0 \implies \text{Proved.}$$

⇒ Symmetrical System.

In a three phase system, three voltages having same magnitude and frequency and displaced from each other by 120° phase angle.

## Star Connection Y




* Phase Voltage ($V_{ph}$)
  ↳ Voltage between Neutral (N) and any one phase R (or) Y (or) B.

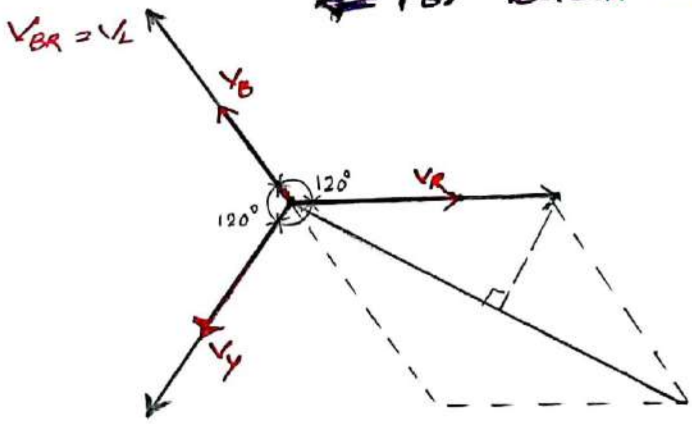
$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

* phase current ⇒ $I_R, I_Y, I_B$.
* Line Current ⇒ $I_{Line}$

$$I_{phase} = I_{Line}$$

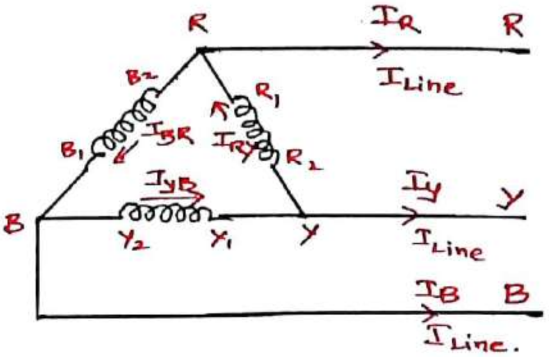
* Power $(P) = \sqrt{3}\, V_L I_L \cos\phi$ W

⟸ For Balanced Load (or) Supply ⟹




* From triangle OAB
$$\cos 30^\circ = \frac{OC}{OA} = \frac{(V_{RY}/2)}{V_R}$$
$$\frac{\sqrt{3}}{2} = \frac{V_{RY}}{2V_R}$$
$$\sqrt{3}\, V_R = V_{RL} \Rightarrow V_L = \sqrt{3}\, V_{ph}.$$

## Delta Connection, △



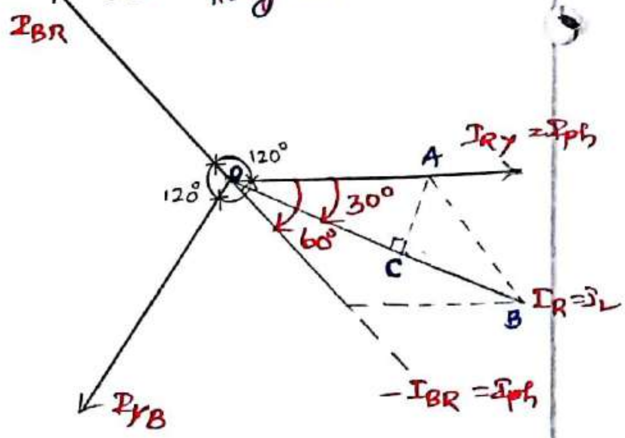

* Line Voltage ($V_L$)
  ↳ Voltage between any two phases (R-Y) or (Y-B) or (B-R).

$$V_L = V_{ph}.$$

* Phase Current $I_R, I_Y, I_B$.
* Line Current $I_{Line}$.

$$I_L = \sqrt{3}\, I_{phase}.$$

* Power $(P) = \sqrt{3}\, V_L I_L \cos\phi$. W.




* From triangle OAB
$$\cos 30^\circ = \frac{B.H}{B} = \frac{OC}{OA}$$
$$\frac{\sqrt{3}}{2} = \frac{I_R/2}{I_{RY}} = \frac{I_R}{2 I_{RY}}$$
$$I_L = I_R = \sqrt{3}\, I_{RY} = \sqrt{3}\, I_{ph}.$$

Prob-1: Three inductive coils each having resistance of 16 Ω and reactance of 12 Ω are connected in star across a 400V, three phase 50Hz supply. Calculate.

* Line Voltage
* phase Voltage
* Line Current
* phase Current
* Power factor
* Power observed
* Draw the phasor diagram.

[AU-Dec.10,11,12 & May-11,12]

Sol

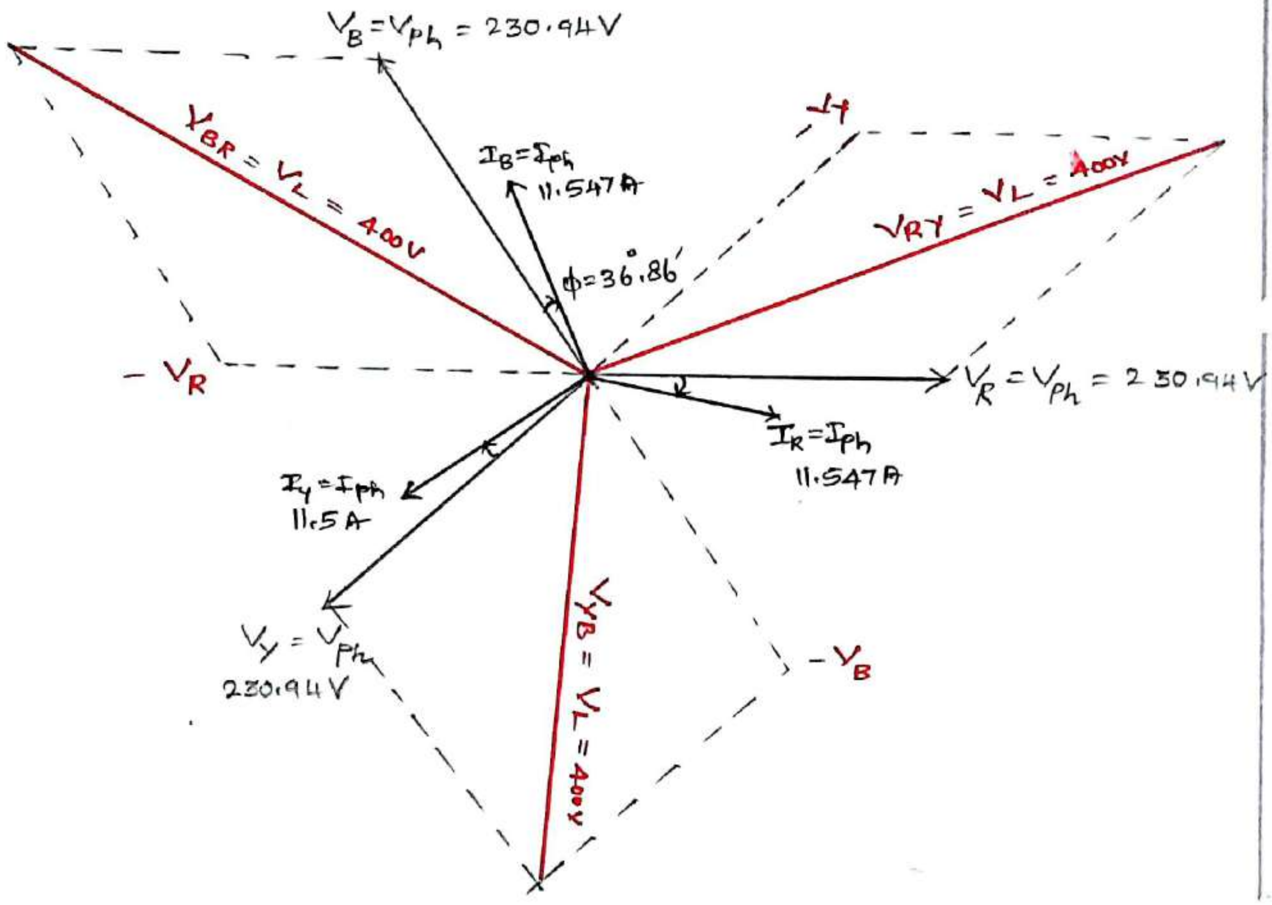
Given data

* Resistance per ph. $(R_{ph}) = 16\,\Omega$
* Reactance per ph $(X_L) = 12\,\Omega$/phase.
* Supply voltage $(V_L) = 400V$

⇒ Impedence per phase $Z_{ph} = R_{ph} + jX_L = (16 + j12)\,\Omega$ → rect

$Z_{ph} = (20 \angle 36.86^\circ)\,\Omega$ → Polar

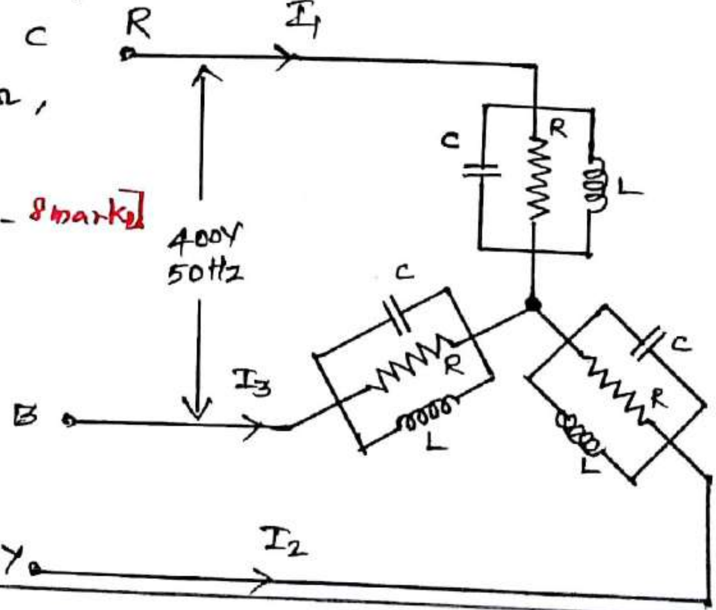
* Line Voltage $(V_L) = 400V$.
* phase Voltage $(V_{ph}) = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$
* phase Current $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{20} = 11.547A$
* Line Current $I_L = I_{ph} = 11.547A$ (For Star conn)
* Power factor $(\cos\phi) = \frac{R_{ph}}{Z_{ph}} = \frac{16}{20} = 0.8$ lagging.
* Power observed $(P) = \sqrt{3}\, V_L I_L \cos\phi = 6400$ watts.




Prob-2. For the circuit shown below, calculate the line current the power and power factor.

The value of R, L and C in each phase are 10 Ω, 1H and 100 μF.

[AU = DEC-10, MAY-12 - 8 marks]



Given data

* Supply Voltage ($V_L$) = 400V
* Resistance/phase ($R_{ph}$) = $10\,\Omega$
* Inductance/phase ($L_{ph}$) = 1H
* Capacitance/phase ($C_{ph}$) = $100\mu F = 100 \times 10^{-6} F$
* Supply frequency ($f$) = 50 Hz.

Sol

* Inductive Reactance/phase ($X_L$) = $2\pi f L = 2 \times \pi \times 50 \times 1$

$$X_L = 314.168\,\Omega$$

* Capacitive Reactance/phase ($X_c$) = $\frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}}$

$$X_c = 31.8309\,\Omega.$$

* Impedence ($Z_{ph}$) is the parallel combination of R, L, C.

$$Z_{eq} \text{ per phase } (Z_{eq}) = \frac{1}{Y_{eq}} = \frac{1}{(Y_1 + Y_2 + Y_3)}$$

↳ Admittance $Y_1 = \frac{1}{R\angle 0^\circ} = \frac{1}{10\angle 0^\circ} = 0.1\angle 0^\circ$

$$Y_1 = (0.1 + j0)\,℧$$

↳ Admittance $Y_2 = \frac{1}{X_L \angle 90^\circ} = \frac{1}{314.16\angle 90^\circ} = 3.18 \times 10^{-3} \angle -90^\circ$

$$Y_2 = (0 - j3.183 \times 10^{-3})\,℧$$

↳ Admittance $Y_3 = \frac{1}{X_c \angle -90^\circ} = \frac{1}{31.83\angle -90^\circ} = (0 + j0.0314)\,℧$

$\Rightarrow Y_{eq} = (Y_1 + Y_2 + Y_3) = (0.1 + j\,0.028227)\,℧$

$$Y_{eq} = 0.1039\angle 15.76\,℧$$

$\Rightarrow$ Impedence $Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{0.1039 \angle 15.76}$

$$Z_{eq} = (9.6246 \angle -15.76)\ \Omega$$

* For Star connection $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$

$$V_{ph} = 230.94 \text{ Volts.}$$

* Current per phase $(I_{ph}) = \frac{V_{ph}}{Z_{ph}}$

$$= \frac{230.94 \angle 0^{\circ}}{9.6246 \angle -15.76}$$

$$I_{ph} = 24 \angle 15.76^{\circ} \text{ Amp.}$$

* For Star connection $I_L = I_{ph} = 24 \angle 15.76$ Amp.

* Power factor angle $\phi = +15^{\circ}.76'$ leading.

* Power absorbed $(P) = \sqrt{3} * V_L * I_L * \cos\phi.$

$$= \sqrt{3} * 400 * 24 * \cos(15.76)$$

$$P = 16.0026 \text{ kW.}$$

* Power factor $(\cos\phi) = \cos(15.76) = 0.9624$ leading.